Math 101 Fall 2004 Exam 1
Instructor: Jennifer Wightman/Richard Stong
Thursday, September 30, 2004

Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have one hour and fifteen minutes. Do all 8 problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work will receive no credit.
Please print you name clearly here.

Print name: __________________________________________

Upon finishing please sign the pledge below:
On my honor I have neither given nor received any aid on this exam.

Grader’s use only:

1. _______/10
2. _______/15
3. _______/10
4. _______/20
5. _______/10
6. _______/10
7. _______/15
8. _______/10
1. [10 points] Evaluate the following limits, if they exist.

(a) \( \lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 4x + 3} \)

(b) \( \lim_{\theta \to 0} \frac{1}{5} \sin \frac{\theta}{3} \)
2. [15 points] Suppose $c$ is a constant and the function $f$ is given by:

$$f(x) = \begin{cases} 
  cx^2, & x < 1 \\
  3cx - 2, & x \geq 1 
\end{cases}$$

(a) Calculate $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$.

(b) Find all values of the constant $c$ so that the function $f$ is continuous everywhere.
3. [10 points] (a) Give the formal, mathematical definition of the derivative of a function $f(x)$ at the point $x = a$.

(b) Find the derivative of $f(x) = \frac{x}{1-x^2}$ using the definition of the derivative. (No credit will be given for finding the derivative by other means.)
4. [20 points] Calculate the derivative for each of the following functions:

(a) \( g(x) = (8x^2 - 5x)(13\sqrt{x} + 4) \)

(b) \( f(x) = \frac{\sec x}{2x^2 - 4x + 8} \)

(c) \( k(\theta) = \cos^2 (e^{3\theta} + 1) \)

(d) \( f(x) = (7x + \ln(x^2))^6 \)
5. [10 points] Find the equation of the tangent line to the graph of $y = xe^{2x}$ at $x = 1$. 
6. [10 points] Find the maximum and minimum value of the function \( f(x) = x\sqrt{2} - x^2 \) on the interval \([-\sqrt{2}, \sqrt{2}]\). Be sure to show all the steps you need to show in order to justify that your answers really are the maximum and minimum.
7. [15 points] A grain silo is to be built in the shape of a right circular cylinder with a hemispherical top. The concrete floor of the silo costs $10 per square foot and the sides and top cost $3 per square foot. You have 3600\pi dollars to spend. What is the maximum possible volume for the silo? (Some useful geometry formulas: The volume of a right circular cylinder of height \( h \) and radius \( r \) is \( V = \pi r^2 h \) and the curved part of the surface of the cylinder has area \( S = 2\pi rh \). The volume and surface area of a sphere of radius \( r \) are \( V = \frac{4}{3}\pi r^3 \) and \( S = 4\pi r^2 \), respectively.)
8. [10 points] Find $dy/dx$ for the following by implicit differentiation.

$$(x^2 + y^2)^2 = 50xy.$$