Math 101 Fall 2002 Exam 1

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Tuesday, October 1, 2002

Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have one hour and fifteen minutes. Do all 7 problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work will receive no credit.

Please print you name clearly here.

Print name: ____________________________________________

Upon finishing please sign the pledge below:

On my honor I have neither given nor received any aid on this exam.

Grader’s use only:

1. _______/15

2. _______/15

3. _______/10

4. _______/20

5. _______/10

6. _______/15

7. _______/15
1. [15 points] Evaluate the following limits, if they exist.
   (a) \( \lim_{x \to -2} \frac{x^2 + x - 2}{x^2 - x - 6} \)
   
   (b) \( \lim_{\theta \to 0} \frac{\tan 2\theta}{\sin 5\theta} \)
   
   (c) \( \lim_{x \to 1} \frac{2x - 1}{x - 1} \)
2. [15 points] Suppose $c$ is a constant and the function $f$ is given by:

$$f(x) = \begin{cases} 
  c^2 - x^2, & x < 0 \\
  2(c - x)^2, & x \geq 0 
\end{cases}$$

(a) Calculate the following limits:

$$\lim_{x \to 0^-} f(x) \quad \text{and} \quad \lim_{x \to 0^+} f(x)$$

(b) Find a value of the constant $c$ so that the function $f$ is continuous everywhere.
3. [10 points] Find the derivative of $f(x) = \sqrt{x+3}$ using the definition of the derivative. (No credit will be given for finding the derivative by other means.)
4. [20 points] Calculate the derivative for each of the following functions:

(a) \((4x^2 + 7x + 3)^{50}\)

(b) \((1 + 2x)^5 \sin(2x^3)\)

(c) \(3 + \frac{2x}{\sqrt{x+1}}\)

(d) \(\cos^2(3e^x)\)
5. [10 points] Find the equation of the tangent line to the graph of \( y = \tan(2x) + 3 \sec x \) at the point \((0, 3)\).
6. [15 points] Find the maximum and minimum value of $f(x) = \frac{1-x}{x^2+3}$ on $[-2, 1]$. Be sure to show all the steps you need to show in order to justify that your answers really are the maximum and minimum.
7. [15 points] A rectangle of perimeter 24 inches is rotated about one of its sides to generate a right circular cylinder. What are the dimensions of the rectangle which give a cylinder of maximal volume? (Recall that the volume of a right circular cylinder with height $h$ and radius $r$ is $V = \pi r^2 h$. )