SYLLABUS - HOLOMORPHIC VECTOR BUNDLES ON RIEMANN SURFACES

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1. Syllabus

- **Week 1**: The topology and complex structure of Riemann surfaces (complex manifolds, Riemann-Hurwitz formula, a plethora of examples)
- **Week 2**: Topological vector bundles (basic concepts such as degree, some concrete constructions, classification of topological vector bundles on Riemann surfaces)
- **Week 3/4**: Holomorphic line bundles. How many sections does a line bundles have? What is a sheaf? What is sheaf cohomology? Why do we care? (one reason, a vector bundle is just a special type of sheaf)
- **Week 5**: More examples and computations of sheaf cohomology (Riemann-Roch for curves)
- **Week 6**: What are all the holomorphic line bundles on a Riemann surface? How does sheaf cohomology help us answer this question?
- **Week 7**: What is a divisor and how is it related to holomorphic line bundles? (Abel’s theorem)
- **Week 8/9**: Classifying holomorphic vector bundles on \( \mathbb{P}^1 \) (Grothendieck’s theorem)
- **Week 10**: What are the holomorphic vector bundles on genus one Riemann surfaces
- **Week 11/12**: Holomorphic vector bundles on curves of genus \( g \geq 2 \) (selected topics trying to build on ideas from the previous section)

2. References

(1) Beauville. Vector Bundles on Curves and Generalized Theta Functions.
(2) J Le Potier. Lectures on Vector Bundles.
(3) Shafarevich. Basic Algebraic Geometry.
(4) Griffiths and Harris. Principles of Algebraic Geometry.

3. Prerequisites

Students should be familiar with basic complex analysis and basic ideas from differential geometry such as that of a manifold.