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Essays on representations of p-adic groups

The Jacquet module

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In this essay, let G be the $\mathfrak{k}$-rational points on a reductive group defined over $\mathfrak{k}$. I'll discuss here representations induced from parabolic subgroups to G, as well as a related construction going from representations of G to those of parabolic subgroups.

These constructions, which are adjoint in a technical sense, turn out to control much of the structure of admissible representations induced from parabolic subgroups, and also to describe the behaviour at infinity on G of admissible matrix coefficients. Parabolic induction is a classic technique in representation theory, but the adjoint construction has its origins in [Jacquet-Langlands:1970] and [Jacquet:1971].

Contents

1. Representations induced from parabolic subgroups

Suppose P to be a parabolic subgroup of G, and (σ, U) an admissible representation of P. The (normalized) representation induced from σ is the right regular representation of G on the space

$$\mathrm{Ind}(\sigma \mid P, G) = \{ f \in C^\infty(G, U) \mid f(pg) = \sigma(p)\delta_P^{1/2}(p) f(g) \text{ for all } p \in P, g \in G \} .$$

Here

$$\delta_P(p) = \bigl|\det\nolimits_{\mathfrak{n}}(p)\bigr| .$$

♣ **[induced-admissible]** Since $P\backslash G$ is compact, according to Proposition 9.1(smooth) this representation is admissible. If K is what Bruhat & Tits call a 'good' maximal compact subgroup (so that $G = PK$), the restriction of $\mathrm{Ind}(\sigma \mid P, G)$ to K is a K-isomorphism of $\mathrm{Ind}(\sigma \mid P, G)$ with $\mathrm{Ind}(\sigma \mid K \cap P, K)$.

An **unramified character** of P is one that is trivial on $K \cap P$. It follows from the observation above that if χ is unramified then $\mathrm{Ind}(\sigma \mid P, G)$ and $\mathrm{Ind}(\sigma\chi \mid P, G)$ are canonically isomorphic as K-representations.

♣ **[frobenius]** Proposition 9.3(smooth) implies:

[parabolic-frob] **Proposition 1.1.** *If (π, V) is an admissible representation of G and (σ, U) one of P, then composition with the map*

$$\Omega_1 \colon f \longmapsto f(1)$$

induces an isomorphism

$$\mathrm{Hom}_G\bigl(V, \mathrm{Ind}(\sigma \mid P, G)\bigr) \cong \mathrm{Hom}_P(V, U) .$$

[induced-dual] **Proposition 1.2.** *The contragredient of* $\mathrm{Ind}(\sigma \mid P, G)$ *is isomorphic to* $\mathrm{Ind}(\widetilde{\sigma} \mid P, G)$*.*

If f lies in the first and F the second, then the product-pairing $\langle f(x), F(x)\rangle$ lies in $\mathrm{Ind}(\delta^{1/2} \mid P, G)$. The pairing is then

$$\int_{P\backslash G} \langle f(x), F(x)\rangle \, dx$$

[induced-dual] **Proposition 1.3.** *The representation* $\mathrm{Ind}(\sigma \mid P, G)$ *is unitary if* σ *is .*

There is a great deal more to be said about these representations, but first we need to investigate admissible representations of P.

2. Admissible representations of parabolic subgroups

Let $P = M_P N_P = MN$ be a parabolic subgroup of G, $A = A_P$ the split centre of M_P. There exists a basis of neighbourhoods of P of the form $U_M U_N$ where U_M is a compact open subgroup of M, U_N is one of N, and U_M conjugates U_N to itself.

The group M may be identified with a quotient of P, and therefore the admissible representations of M may be identified with those of P trivial on N. It happens that there are no others:

[parabolic-admissible] **Proposition 2.1.** *Every admissible representation of* P *is trivial on* N*.*

Proof. Let (π, V) be an admissible representation of P over the field $\mathcal{F}$, and suppose v in V. We want to show that $\pi(n)v = v$ for all n in N.

So suppose n in N, and suppose $U = U_M U_N$ fixes v. We can find a in A such that $a^{-1}na \in U_N$ as well as $a^{-1}Ua \subseteq U$. Then $\pi(a)$ takes V^U to itself, since for u in U we have

$$\pi(u)\pi(a)v = \pi(a)\pi(a^{-1}ua)v = \pi(a)v\,.$$

The operator $\pi(a)$ is certainly injective. I shall prove in a moment that it is bijective on V^U. Assuming that $\pi(a)v_* = v$:

$$\begin{aligned}\pi(n)v &= \pi(n)\pi(a)\pi(a^{-1})v \\ &= \pi(n)\pi(a)v_* \\ &= \pi(a)\pi(a^{-1}na)v_* \\ &= \pi(a)v_* \\ &= v\,.\end{aligned}$$

If $\mathcal{F}$ is a field, the claim that $\pi(a)$ is surjective is is trivial, since V^U is finite-dimensional and $\pi(a)$ is injective. But if $\mathcal{F}$ is an arbitrary commutative Noetherian ring, things are more difficult. We begin with a preliminary result.

[noetherian] **Lemma 2.2.** *Suppose* B *to be a finitely generated module over the Noetherian ring* R*. If* $f\colon B \to B$ *is an* R*-injection with the property that for each maximal ideal* $\mathfrak{m}$ *of* R *the induced map* $f_{\mathfrak{m}}\colon B/\mathfrak{m}B \to B/\mathfrak{m}B$ *is also injective, then* f *is itself an isomorphism.*

Proof. Let C be the quotient $B/f(B)$. The exact sequence

$$0 \longrightarrow B \xrightarrow{f} B \longrightarrow C \longrightarrow 0$$

induces for each $\mathfrak{m}$ an exact sequence

$$0 \longrightarrow B/\mathfrak{m}B \xrightarrow{f_{\mathfrak{m}}} B/\mathfrak{m}B \longrightarrow C/\mathfrak{m}C \longrightarrow 0\,.$$

It is by assumption that the left hand map is injective. Since $F = R/\mathfrak{m}$ is a field and B is finitely generated, the space $B/\mathfrak{m}B$ is a finite-dimensional vector space over F, and therefore $f_{\mathfrak{m}}$ an isomorphism. Hence $C/\mathfrak{m}C = 0$ for all $\mathfrak{m}$. The module C is Noetherian, which means that if $C \neq 0$, it possesses at least one maximal proper submodule D. The quotient C/D must be isomorphic to $R/\mathfrak{m}$ for some maximal ideal $\mathfrak{m}$. But then $C/\mathfrak{m}C \neq 0$, a contradiction. Therefore $C = 0$ and f an isomorphism. ∎

As before, we must show that if (π, V) is an admissible representation of P and v in V, then $\pi(n)v = v$ for all n in N.

♣ [parabolic-admissible] To return to the proof of Proposition 2.1, the Lemma assures us that $\pi(a)$ is surjective, hence bijective. ∎

3. The Jacquet module

Suppose $P = MN$ a parabolic subgroup of G. Suppose (π, V) an admissible representation of G, (σ, U) ♣ [frobenius] one of P. Frobenius reciprocity (Proposition 9.3(smooth)) tells us that

$$\mathrm{Hom}_G\big(V, \mathrm{Ind}(\sigma \mid P, G)\big) \cong \mathrm{Hom}_P(V, U)\,,$$

while the results of the previous section tell us that σ is trivial on N and factors through the canonical projection $P \to M$. In this section we explore the consequences of joining these two facts.

[unipotent-large] **Lemma 3.1.** *If N is a $\mathfrak{p}$-adic unipotent group, it possesses arbitrarily large compact open subgroups.*

Proof. It is certainly true for the group of unipotent upper triangular matrices in GL_n. Here, if a is the diagonal matrix with $a_{i,i} = \varpi^i$ then conjugation by powers of a will scale any given compact open subgroup to an arbitrarily large one. But any unipotent group can be embedded as a closed subgroup in one of these. ∎

Fix the parabolic subgroup $P = MN$. If (π, V) is any smooth representation of N, define $V(N)$ to be the subspace of V generated by vectors of the form

$$\pi(n)v - v$$

as n ranges over N. The group N acts trivially on the quotient

$$V_N = V/V(N)$$

It is universal with respect to this property:

[universality] **Proposition 3.2.** *The projection from V to V_N induces for every smooth $\mathcal{F}$-representation (σ, U) on which N acts trivially an isomorphism*

$$\mathrm{Hom}_N(V, U) \cong \mathrm{Hom}_{\mathcal{F}}(V_N, U)\ .$$

[union-vu] **Lemma 3.3.** *For v in V the following are equivalent:*

- *(a) v lies in $V(N)$;*
- *(b) v lies in $V(U)$ for some compact open subgroup of N;*
- *(c) we have*

$$\int_U \pi(u)v\,du = 0$$

for some compact open subgroup U of N.

♣ [unipotent-large] ♣ [projection] *Proof.* The equivalence follows immediately from Lemma 3.1. That of (b) and (c) follows from Proposition 1.1(smooth) . ∎

[jacquet-exact] **Proposition 3.4.** *If*

$$0 \to U \to V \to W \to 0$$

is an exact sequence of smooth representations of N*, then the sequence*

$$0 \to U_N \to V_N \to W_N \to 0$$

is also exact.

Proof. That the sequence

$$U_N \to V_N \to W_N \to 0$$

is exact follows immediately from the definition of $V(N)$. The only non-trivial point is the injectivity of $U_N \to V_N$. If u in U lies in $V(N)$ then it lies in $V(S)$ for some compact open subgroup S of N.
♣ [projection] According to Proposition 1.1(smooth) , the space V has a canonical decomposition

$$V = V^S \oplus V(S) \; ,$$

and v lies in $V(S)$ if and only if

$$\int_S \pi(s) v \, ds = 0$$

But this last equation holds in U as well, since U is stable under S, so v must lie in $U(S)$. □

If σ is trivial on N, any P-map from V to U factors through V_N. The space $V(N)$ is stable under P, and there is hence a natural representation of M on V_N. The **Jacquet module** of π is this representation twisted by the character $\delta_P^{-1/2}$. In other words, if u lies in $V/V(N)$ and v in V has image u, then

$$\pi_N(m)u \;\text{ is the image of } \delta_P^{-1/2}(m)\pi(m)v$$

for m in M. This is designed exactly to allow the simplest formulation of this:

[jacquet-frobenius] **Proposition 3.5.** *If* (π, V) *is any smooth representation of* G *and* (σ, U) *one of* M *then evaluation at* 1 *induces an isomorphism*

$$\mathrm{Hom}_G\big(\pi, \mathrm{Ind}(\sigma \,|\, P, G)\big) \cong \mathrm{Hom}_M(\pi_N, \sigma)$$

4. Iwahori factorizations

Suppose $P = MN$ to be a parabolic subgroup of G, $\overline{P}$ the corresponding opposite one, with $\overline{P} \cap P = M$. If K is a compact open subgroup, it is said to have an **Iwahori factorization** with respect to P if (a) the product map from $\overline{N}_K \times M_K \times N_K$ to K is a bijection, where $M_K = M \cap K$ etc. and (b) $aN_Ka^{-1} \subset N_K$, $a^{-1}\overline{N}_K a \subset \overline{N}_K$ for every a in A_P^{--}.

[iwahori-factorization] **Lemma 4.1.** *Let* $P_\emptyset$ *be a minimal parabolic subgroup of* G*. There exists a basis* $\{K_n\}$ *of neighbourhoods of* $\{1\}$ *in* G *such that*

- *(a) each* K_n *is a normal subgroup of* K_0*;*
- *(b) If* P *is a parabolic subgroup of* G *containing* $P_\emptyset$ *then* K_n *has an Iwahori factorization with respect to* P*;*
- *(c) if* $P = MN$ *is a parabolic subgroup containing* $P_\emptyset$ *then* $M \cap K_n$ *has an Iwahori factorization with respect to* $M \cap P_\emptyset$*.*

Proof. Assume first that G is split over $\mathfrak{k}$. According to [Iwahori-Matsumoto:1967], there exists a split group scheme $G_{\mathfrak{o}}$ defined over $\mathfrak{o}$ with $G = G_{\mathfrak{o}} \times_{\mathfrak{o}} \mathfrak{k}$ (i.e. defined by base extension from $\mathfrak{o}$ to $\mathfrak{k}$). We may choose $P_\emptyset$ also obtained by base extension from a smooth parabolic subgroup defined over $\mathfrak{o}$. The sequence of congruence subgroups $G(\mathfrak{p}^n)$ satisfies the conditions of the Lemma.

Now suppose G arbitrary. Let $\mathfrak{l}/\mathfrak{k}$ be a finite Galois extension $\mathfrak{l}/\mathfrak{k}$ over which G splits. Let $K_{\mathfrak{l},n}$ be a sequence satisfying the Proposition for a minimal parabolic subgroup contained in $P_\emptyset \times_{\mathfrak{k}} \mathfrak{l}$, and let $K_n = K_{\mathfrak{l},n} \cap G$. Galois theory together with uniqueness of the Iwahori factorizations allows us to conclude. □

5. Admissibility of the Jacquet module

Now fix an admissible representation (π, V) of G. Let $P, \overline{P}$ be an opposing pair of parabolic subgroups, K_0 to be a compact open subgroup possessing an Iwahori factorization $K_0 = N_0 M_0 \overline{N}_0$ with respect to this pair. For each a in A_P^{--} let T_a be the smooth distribution $\mu_{K_0 a K_0/K_0}$ on G. For any smooth representation (π, V) and v in V^{K_0} let τ_a be the restriction of $\pi(T_a)$ to V^{K_0}. Thus for v in V^{K_0}

$$\begin{aligned} \tau_a(v) &= \pi(T_a)\, v \\ &= \sum_{K_0 a K_0/K_0} \pi(g)\, v \\ &= \sum_{K_0/K_0 \cap a K_0 a^{-1}} \pi(k)\pi(a)\, v\,. \end{aligned}$$

This is valid since the isotropy subgroup of a in the action of K_0 acting on $K_0 a K_0/K_0$ is $aK_0a^{-1} \cap K_0$, hence

$$k \mapsto kaK_0$$

is a bijection of $K_0/K_0 \cap aK_0a^{-1}$ with K_0aK_0/K_0.

[jprojection] **Lemma 5.1.** *If v lies in V^{K_0} with image u in V_N, the image of $\tau_a v$ in V_N is equal to $\delta_P^{-1/2}(a)\pi_N(a)u$.*

Proof. Since $K_0 = N_0 M_0 \overline{N}_0$, $aK_0a^{-1} = (aN_0a^{-1})M_0(aN_0a^{-1})$. Since $\overline{N} \subseteq a\overline{N}a^{-1}$, the inclusion of N_0/aN_0a^{-1} into $K_0/(aK_0a^{-1} \cap K_0)$ is in turn a bijection. Since the index of aN_0a^{-1} in N_0 or, equivalently, that of N_0 in $a^{-1}N_0a$ is $\delta_P^{-1}(a)$:

$$\begin{aligned} \tau_a(v) &= \sum_{K_0/aK_0a^{-1} \cap K_0} \pi(k)\pi(a)v \\ &= \sum_{N_0/aN_0a^{-1}} \pi(n)\pi(a)v \\ &= \pi(a) \sum_{a^{-1}N_0a/N_0} \pi(n)v\,. \end{aligned}$$

Since $\pi(n)v$ and v have the same image in V_N, and $[a^{-1}N_0a\colon N_0] = \delta_P^{-1}(a)$, this concludes the proof. □

[tab] **Lemma 5.2.** *For every a, b in A_P^{--},*

$$\tau_{ab} = \tau_a \tau_b$$

Proof. We have

$$\begin{aligned} T_a T_b &= \sum_{N_0/aN_0a^{-1}} \sum_{N_0/bN_0b^{-1}} \pi(n_1\pi(a)\pi(n_2)\pi(b)v \\ &= \sum_{N_0/aN_0a^{-1}} \sum_{N_0/bN_0b^{-1}} \pi(n_1)\pi(an_2a^{-1})\pi(ab)v \\ &= \sum_{N_0/abN_0b^{-1}a^{-1}} \pi(n)\pi(ab)v \\ &= T_{ab} \end{aligned}$$

since as n_1 ranges over representatives of N_0/aN_0a^{-1} and and n_2 over representatives of N_0/bN_0b^{-1}, the products $n_1\, an_2a^{-1}$ range over representatives of $N_0/abN_0b^{-1}a^{-1}$. □

[kernel-ta] **Lemma 5.3.** *For any a in A_P^{--} the subspace of V^{K_0} on which τ_a acts nilpotently coincides with $V^{K_0} \cap V(N)$.*

Proof. Since $\mathcal{F}$ is Noetherian and V^{K_0} finitely generated, the increasing sequence

$$\ker(\tau_a) \subseteq \ker(\tau_{a^2}) \subseteq \ker(\tau_{a^3}) \subseteq \ldots$$

is eventually stationary. It must be shown that it is the same as $V^{K_0} \cap V(N)$.

Choose n large enough so that $V^{K_0} \cap V(N) = V^{K_0} \cap V(a^{-n}N_0a^n)$. Let $b = a^n$. Since

$$\tau_b v = \pi(b) \sum_{b^{-1}N_0b/N_0} \pi(n)v \ ,$$

and $\tau_b v = 0$ if and only if $\sum_{b^{-1}N_0b/N_0} \pi(n)v = 0$, and again if and only if v lies in $V(N)$. □

The canonical map from V to V_N takes V^{K_0} to $V_N^{M_0}$. The kernel of this map is $V \cap V(N)$, which by
♣ [kernel-ta] Lemma 5.3 is equal to the kernel of τ_{a^n} for large n.

[stable] **Lemma 5.4.** *The image of τ_{a^n} in V^{K_0} is independent of n if n is large enough. The map τ_a is invertible on it. The intersection of it with $V(N)$ is trivial.*

♣ [kernel-ta] *Proof.* Choose n so large that $\ker(\tau_{a^n}) = \ker(\tau_{a^m})$ for all $m \geq n$. By Lemma 5.3 this kernel coincides with $V^{K_0} \cap V(N)$. Let U be the image of τ_{a^n}. If $u = \tau_{a^n}v$ and $\tau_{a^n}v = 0$ then $\tau_{a^{2n}}v = 0$, which means by assumption that in fact $u = \tau_{a^n}v = 0$. Therefore the intersection of U with $V(N)$ is trivial, the projection from V to V_N is injective on U, and τ_a is also injective on it. If $\mathfrak{m}$ is a maximal ideal of $\mathcal{F}$, this remains
♣ [noetherian] true for $U/\mathfrak{m}U$, and therefore by Lemma 2.2 τ_a is invertible on U. This implies that U is independent of the choice of n. □

Let $V_N^{K_0}$ be this common image of the τ_{a^n} for large n. The point is that it splits the canonical projection from V^{K_0} to $V_N^{M_0}$, which turns out to be a surjection.

[jacquetdecomp] **Proposition 5.5.** *The canonical projection from $V_N^{K_0}$ to $V_N^{M_0}$ is an isomorphism.*

Proof. Suppose given u in $V_N^{M_0}$. Since M_0 is compact, we can find v in V^{M_0} whose image in V_N is u. Suppose that v is fixed also by $\overline{N}_*$ for some small $\overline{N}_*$. If we choose b in A_P^{--} such that $b\overline{N}_0b^{-1} \subseteq \overline{N}_*$, then $v_* = \delta^{1/2}(b)\pi(b)v$ is fixed by $M_0\overline{N}_0$. Because $K_0 = N_0M_0\overline{N}_0$, the average of $\pi(n)v_*$ over N_0 is the same as the average of $\pi(k)v_*$ over K_0. This average lies in V^{K_0} and has image $\pi_N(b)u$ in V_N. But then $\tau_a v_*$ has image $\delta^{1/2}(a)\pi_N(ab)u$ in V_N and also lies in $V_N^{K_0}$. Since τ_{ab} acts invertibly on $V_N^{K_0}$, we can find v_{**} in $V_N^{K_0}$ such that $\tau_{ab}v_{**} = \tau_a\tau_b v_{**} = \tau_a v_*$, and whose image in V_N is u. □

As a consequence:

[jacquet-admissible] **Theorem 5.6.** *If (π, V) is an admissible representation of G then (π_N, V_N) is an admissible representation of M.*

Thus whenever K_0 is a subgroup possessing an Iwahori factorization with respect to P, we have a canonical subspace of V^{K_0} projecting isomorphically onto V^{M_0}. For a given M_0 there may be many different K_0 suitable; how does the space $V_N^{K_0}$ vary with K_0?

[coherence] **Lemma 5.7.** *Let $K_1 \subseteq K_0$ be two compact open subgroups of G possessing an Iwahori factorization with respect to P. If v_1 in $V_N^{K_1}$ and v_0 in $V_N^{K_0}$ have the same image in V_N, then $\pi(\mu_{K_0/K_0})v_1 = v_0$.*

6. The canonical pairing

Continue to let K_0 be a compact open subgroup of G possessing an Iwahori factorization $\overline{N}_0 M_0 N_0$ with respect to the parabolic subgroup P, (π, V) an admissible representation of G.

[annihilation] **Lemma 6.1.** *For v in $V_N^{K_0}$, $\widetilde{v}$ in $\widetilde{V}^{K_0} \cap \widetilde{V}(\overline{N})$, $\langle \widetilde{v}, v \rangle = 0$.*

Proof. Fix a in A^{--} for the moment and choose v_0 in $V_N^{K_0}$ with $\tau_a v_0 = v$. Then

$$\langle v, \widetilde{v} \rangle = \langle \tau_a v_0, \widetilde{v} \rangle = \langle v_0, \tau_{a^{-1}} \widetilde{v} \rangle .$$

♣ [kernel-ta] According to Lemma 5.3(jacquet) , τ_a is nilpotent on $\widetilde{V}^{K_0} \cap \widetilde{V}(\overline{N})$, so if we choose a suitably the right-hand side is 0. □

[asymptotic-pairing] **Theorem 6.2.** *If $(\pi, V$ is an admissible representation of G, then there exists a unique pairing between V_N and $\widetilde{V}_{\overline{N}}$ with the property that whenever v has image u in V_N and $\widetilde{v}$ has image $\widetilde{u}$ in $\widetilde{V}_{\overline{N}}$, then for all a in A_P^{--} near enough to 0*

$$\langle \widetilde{v}, \pi(a) v \rangle = \delta_P^{1/2}(a) \langle \widetilde{u}, \pi_N(a) u \rangle \ .$$

Similarly with the roles of V and $\widetilde{V}$ reversed.

Proof. Let u in V_N and $\widetilde{u}$ in $\widetilde{V}_{\overline{N}}$ be given. Suppose that u and $\widetilde{u}$ are both fixed by elements of M_0. Let v be a vector in $V_N^{K_0}$ with image u, and similarly for $\widetilde{v}$ and $\widetilde{u}$. Define the pairing by the formula

$$\langle \widetilde{u}, u \rangle_{\mathrm{can}} = \langle \widetilde{v}, v \rangle \ .$$

♣♣ [annihilation] [coherence] It follows from Lemma 6.1 and Lemma 5.7 that this definition depends only on u and $\widetilde{u}$, and not on the choices of v and $\widetilde{v}$. That

$$\langle \widetilde{v}, \pi(a) v \rangle = \delta_P^{1/2}(a) \langle \widetilde{u}, \pi_N(a) u \rangle_{\mathrm{can}}$$

♣♣ [annihilation] [coherence] also follows from Lemma 6.1 and Lemma 5.7. That this property characterizes the pairing follows from the invertibility of τ_a on $V_N^{K_0}$. □

This pairing is called the **canonical pairing**.

[dualvn] **Corollary 6.3.** *If $\mathcal{F}$ is a field, the contragredient of (π_N, V_N) is isomorphic as a representation of M_P to $(\pi_{\overline{N}}, \widetilde{V}_{\overline{N}})$.*

Proof. The canonical pairing is invariant under M because for m in M the pairing $\langle \pi_N(m) u, \pi_{\overline{N}}(m) \tilde{u} \rangle_{\mathrm{can}}$ also satisfies the conditions characterizing the canonical pairing.

For non-degeneracy, suppose u in V_N such that $\langle u, \tilde{u} \rangle_{\mathrm{can}} = 0$ for all $\tilde{u} \in \widetilde{V}_{\overline{N}}$. Let v be a canonical lift of u. Let $\tilde{v}$ be arbitrary in $\widetilde{V}^{K_0}$. Suppose $v = \tau_a v_0$ for v_0 also in $V_N^{K_0}$. Then

$$\langle v, \tilde{v} \rangle = \langle \tau(a) v_0, \tilde{v} \rangle = \langle v_0, \tau(a)^{-1} \tilde{v} \rangle .$$

But if we choose a suitably then $\tau_{a^{-1}} v$ lies in $\widetilde{V}_{\overline{N}}$, so this last is a canonical a pairing, hence 0. Therefore $\langle v, \tilde{v} \rangle = 0$ for all $\tilde{v}$ in $\widetilde{V}^{K_0}$, and $v = 0$. □

7. References

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