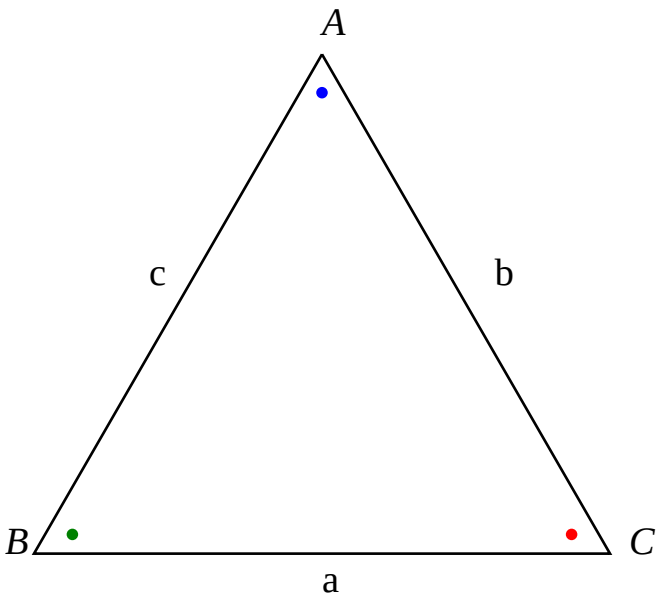
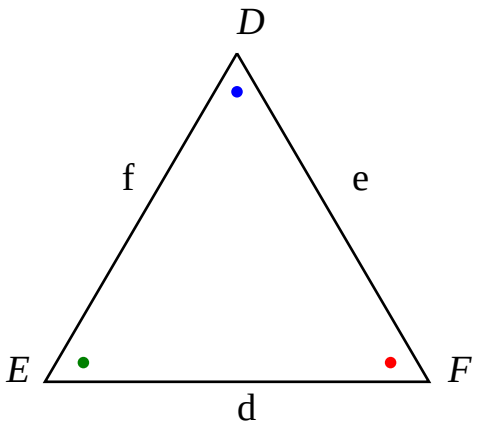
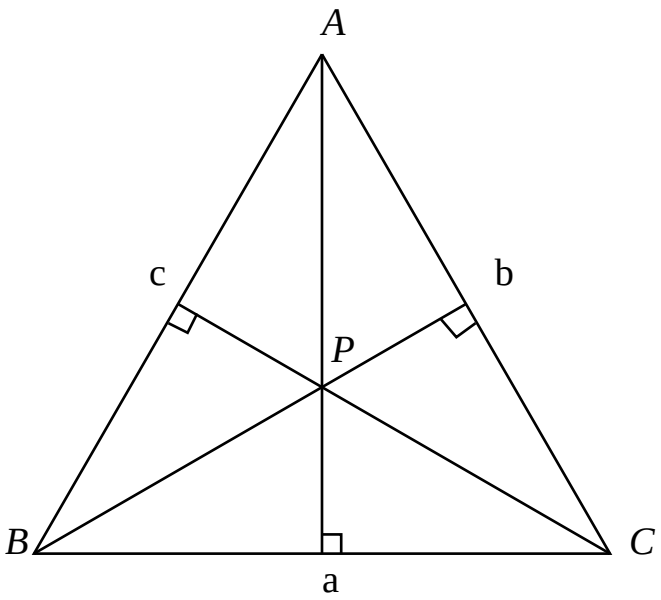
Sim AAA: If two triangles ABC and DEF have their three angles respectively equal, then the corresponding sides are proportional.

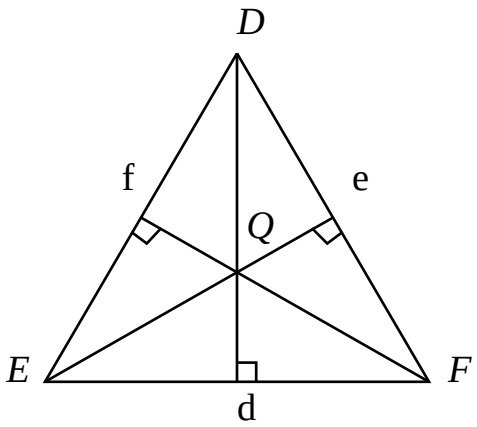




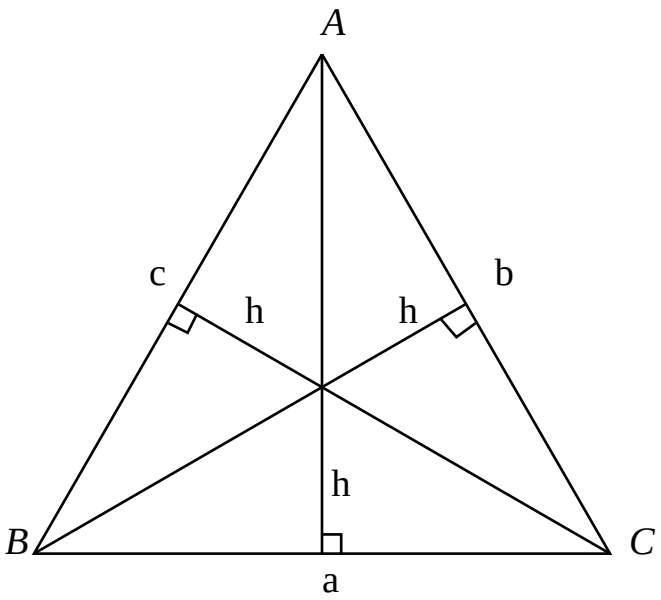
Claim: $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

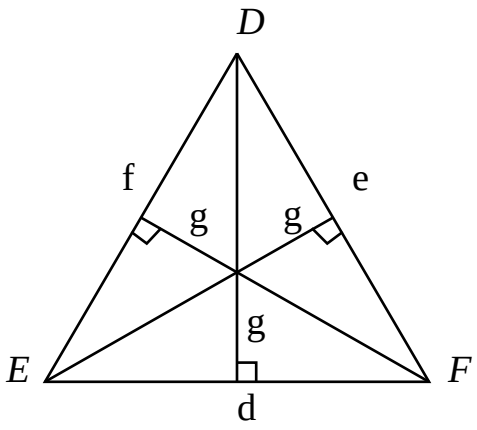
Draw angle bisectors



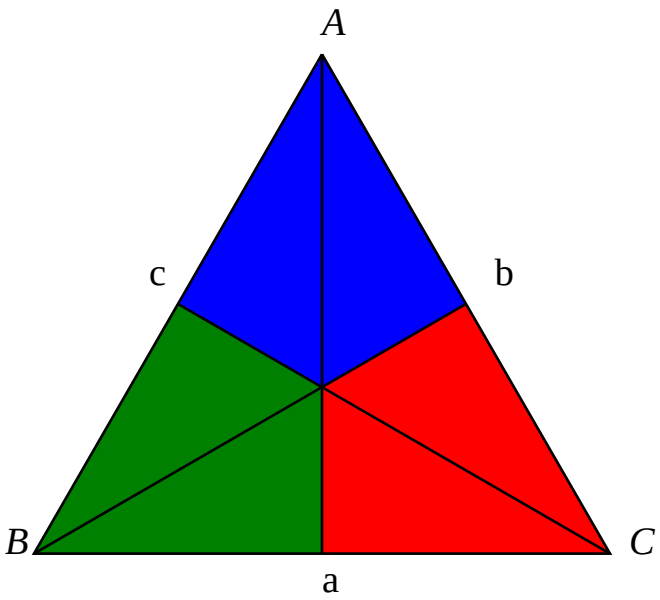


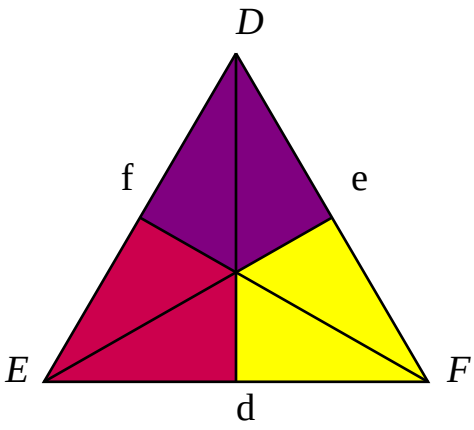
The intersection of the three bisectors are equidistant from the three sides.

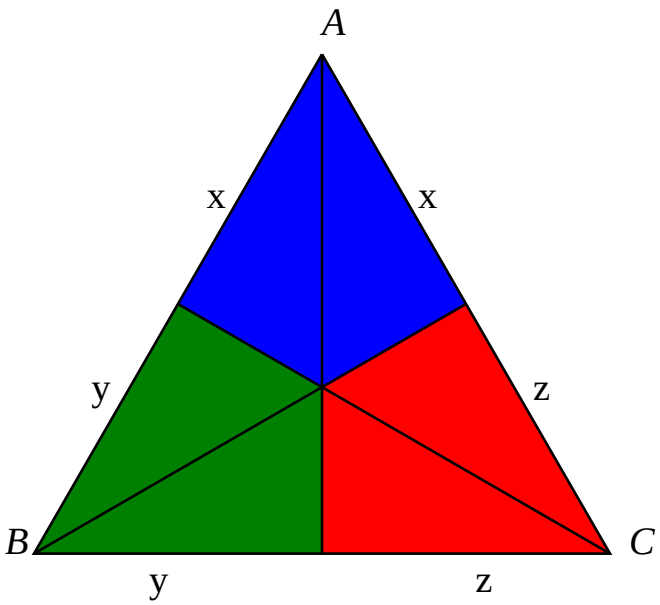


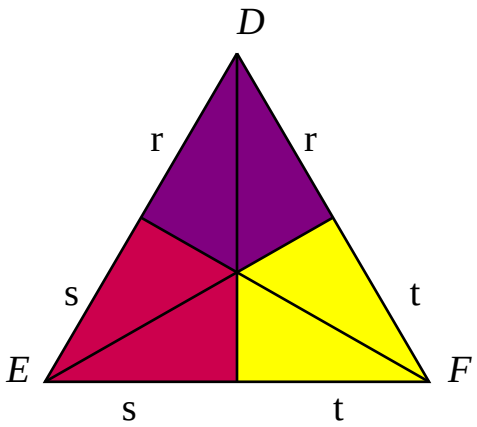


Triangles of the same colour are congruent
SAS

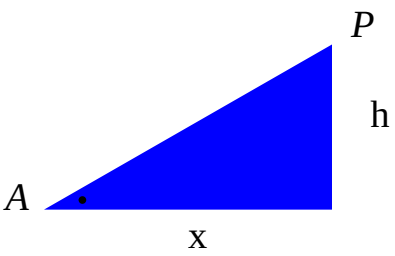


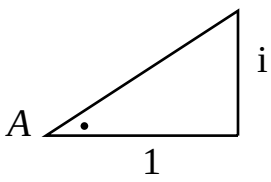



A
x
x
y
z
B
C
y
z

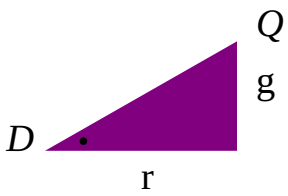

D
r
r
s
t
E
F
s
t

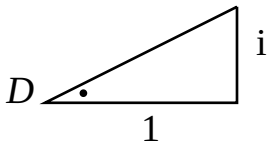
From field segment arithmetic,





$$h = ix$$





$$g = ir$$

Solving for i yields

$$\frac{g}{r} = \frac{h}{x} = i$$

Thus, $\frac{g}{h} = \frac{r}{x}$

Similarily,

$$\frac{s}{y} = \frac{g}{h}$$

$$\frac{t}{z} = \frac{g}{h}$$

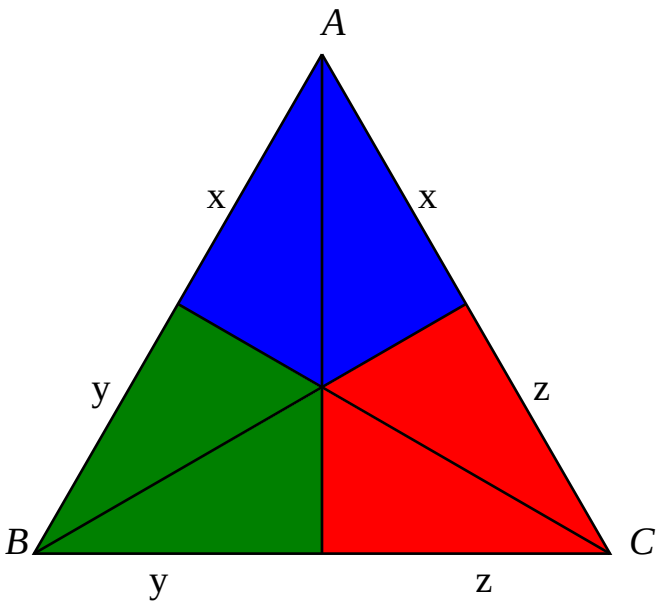
Let $\frac{g}{h} = k$

Then,

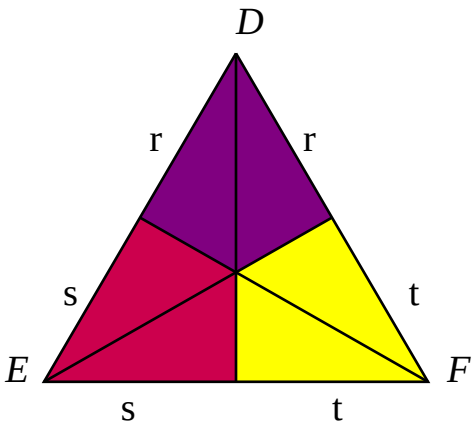
$$x = kr$$

$$y = ks$$

$$z = kt$$



a = y + z



d = s + t

Thus, a = kd

Similarily,

$$b = ke$$

$$c = kf$$

Thus,

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f} = k$$

Q.E.D.