Lecture 12

THE INNER PRODUCT FORMULA

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12.1. The constant term of Eisenstein Series.

We consider a standard parabolic subgroup $P = MN$ and a smooth function ψ on G such that

(i) $\psi(nx) = \psi(x)$ $\qquad n \in \mathbf{N},\ x \in \mathbf{G}$.

(ii) $m \longmapsto \psi(mx)$ is for all $x \in \mathbf{G}$ a square integrable automorphic form on $M\backslash\mathbf{M}^1$ which is a matrix coefficient of some unitary representation π of M with central character ω_π trivial on $A_P(\mathbf{R})^o$.

(iii) $k \longmapsto \psi(xk)$ is K-finite.

We shall say that ψ is cuspidal on P if $m \longmapsto \psi(mx)$ is cuspidal (for all x).

Given $x \in \mathbf{G}$ we have defined $H(x) \in \mathfrak{a}_0$; it may be convenient to introduce its exponential in $A_0(\mathbf{R})^o$:

$$a(x) = \exp H(x)$$

so that for $\lambda \in \mathfrak{a}_0^* \otimes \mathbf{C}$ we may write

$$a(x)^\lambda = e^{\lambda(H(x))} \quad .$$

Let ρ or ρ_P denote the half sum of positive roots of A in N; we have (with notations introduced earlier)

$$\delta_P(x) = a(x)^{2\rho} \quad .$$

Given $\lambda \in \mathfrak{a}_P^* \otimes \mathbf{C}$ and ψ as above, we define

$$\psi(x, \lambda) = \psi(x)a(x)^{\lambda}$$

and if Q is a parabolic subgroup of G containing P we introduce

$$E_Q(x, \psi, \lambda) = \sum_{\gamma \in P\backslash Q} \psi(\gamma x, \lambda+\rho)$$

a convergent series where $\mathrm{Re}(\lambda, \alpha) > (\alpha, \rho)$ for all $\alpha \in \Delta_P^Q$. The left-hand side is known to have a meromorphic analytic continuation to the whole space $\mathfrak{a}_P^* \otimes \mathbf{C}$. We need the formula giving the constant term of E_Q along a parabolic subgroup $R \subset Q$:

$$E_Q^R(x, \psi, \lambda) = \int_{\textcircled{N_R}} E_Q(nx, \psi, \lambda)\,dn \quad .$$

We assume that $\mathrm{Re}(\alpha, \lambda) > (\alpha, \rho)$ for all $\alpha \in \Delta_P^Q$ and then this equals the sum over $\overline{w} \in P\backslash Q/R$ of

$$\int_{\textcircled{N_R}} \sum_{\xi \in R(P,w)} \psi(w\xi nx, \lambda+\rho)\,dn$$

where

$$R(P, w) = R \cap w^{-1}Pw \backslash R \quad .$$

$w_s \in \Omega^R \backslash \Omega^Q/\Omega^P$

$s^{-1}\alpha > 0$ $\alpha \in \Delta_0^R$

$s\alpha > 0$ $\alpha \in \Delta_0^P$

$\Rightarrow s(\Delta_0^P) \in \Delta_0^G$

Then $N_R \cap w_s M w_s^{-1} = \{1\} \Rightarrow w_s M w_s^{-1} \subset M_R$.

We shall assume now that ψ is cuspidal on P and hence the contribution of a double coset $\overline{w} \in P\backslash Q/R$ is zero except perhaps if $wN_R w^{-1} \cap M$ is trivial. In such a case we may assume that the representative w of the double coset $\overline{w}$ is so chosen that $w = w_s^{-1}$ where w_s represents $s \in \Omega^Q$ (the Weyl group of M_Q) satisfying the following properties: $w_s M w_s^{-1} \subset M_R$ and $s^{-1}\alpha > 0$ for all $\alpha \in \Delta_0^R$. Then there exist a standard parabolic subgroup of R which we shall denote by $s\cdot P$ with Levi subgroup $sM = w_s M w_s^{-1}$. The set of all such s will be denoted by $\Omega^Q(\mathfrak{a}_P, R)$. *Why?

Notice that sP depends on s alone and P and not on R.

Let us introduce a coset

$$N^s = N_R \cap w_s N w_s^{-1} \backslash N_R$$

$$= sN \cap w_s N w_s^{-1} \backslash sN$$

$sN = N_R \cdot (sN \cap M_R)$

$w_s^{-1}(N_0 \cap M_R) w_s \subseteq N_0$

$w_s N_0 w_s^{-1} \supset N_0 \cap M_R$.

and define

$$(M(s, \lambda)\psi)(x) = a(x)^{-s\lambda - \rho_s} \int_{\mathbb{N}^s} \psi(w_s^{-1} nx, \lambda + \rho)\, dn$$

where $\rho_s = \rho_{sP}$; the integral is absolutely convergent if $\mathrm{Re}(\lambda, \alpha) > (\rho, \alpha)$ for all $\alpha \in \Delta_P^Q$. In the contribution of $w = w_s^{-1}$ the integral over N_R may be replaced by an integral over N^s and since $R(P, w_s^{-1})/N^s = sP\backslash R$ we have obtained the

LEMMA 12.1.1. <u>Assume that</u> $\mathrm{Re}(\lambda, \alpha) > (\rho, \alpha)$ <u>for all</u> $\alpha \in \Delta_P^Q$ <u>and</u> ψ <u>is cuspidal on</u> P <u>then</u>

The condition that sP exist, i.e. there is the Levi factor of a standard parabolic is a condition on w_s. Condition

* Consider $w_s P w_s^{-1} \cap M_R$. It contains $w_s M w_s^{-1}$. It also contains $N_0 \cap M_R$ because $w_s^{-1}(N_0 \cap M_R) w_s \subseteq N_0$. Thus $w_s P w_s^{-1} \cap M_R$ is a parabolic subgroup of M_R.

$$E_Q^R(x, \psi, \lambda) = \sum_{s \in \Omega^Q(\mathfrak{a}_P, R)} \sum_{\xi \in sP\backslash R} (M(s, \lambda)\psi)(\xi x, s\lambda+\rho_s) \quad .$$

We need a formula for

$$\Lambda^{T,P_1} E_{P_1}(x, \psi, \lambda) = \sum_{R \subset P_1} (-1)^{a_R - a_1}$$

$$\sum_{\delta \in R\backslash P_1} \hat{\tau}_R^{P_1}(H(\delta x)-T) E_{P_1}^R(\delta x, \psi, \lambda) \quad .$$

We shall assume as above that $\mathrm{Re}(\lambda, \alpha) > (\rho, \alpha)$ for all $\alpha \in \Delta_P^{P_1}$ and ψ is cuspidal on P. We see that $\Lambda^{T,P_1} E_{P_1}(x, \psi, \lambda)$ is the sum over $R \subset P_1$ of the sum over $s \in \Omega^{P_1}(\mathfrak{a}_P, R)$ of

$$\sum_{\delta \in sP\backslash P_1} (-1)^{a_R - a_1} \hat{\tau}_R^{P_1}(H(\delta x)-T)(M(s, \lambda)\psi)(\delta x, s\lambda+\rho_s) \quad .$$

As in Lectures 5 and 9 it is convenient to introduce the Weyl sets $\Omega^{P_1}(\mathfrak{a}_P)$ which are the union of the $\Omega^{P_1}(\mathfrak{a}_P, \mathfrak{a}_{P_2})$ for all P_2 standard in P_1. Given $s \in \Omega^{P_1}(\mathfrak{a}_P)$ we define a function on $\mathfrak{a}_P^{P_1}$

$$B_{P|P_1}^s(X) = \sum_{sP \subset R \subset P_1 \cap P_s} (-1)^{a_R} \hat{\tau}_R^{P_1}(sX)$$

when is $s \in \Omega^{P_1}(\mathfrak{a}_P, R)$?
$\Leftrightarrow$ $s^{-1}\alpha > 0$ $\alpha \in \Delta_0^R$
$s\alpha > 0$ $\alpha \in \Delta_0^P$
$\Leftrightarrow$ $s\alpha > 0$ $\alpha \in \Delta_0^P$
$\Delta_0^R \subseteq \Delta_0^{P_s}$

where P_s is the standard parabolic subgroup such that $\alpha \in \Delta_0^{P_s}$ if and only if $s^{-1}\alpha > 0$. We obtain the

LEMMA 12.1.2. <u>Assume</u> $\mathrm{Re}(\lambda, \alpha) > (\rho, \alpha)$ <u>for all</u> $\alpha \in \Delta_P^{P_1}$ <u>and</u> ψ

cuspidal on P then $\Lambda^{T,P_1} E_{P_1}(x, \psi, \lambda)$ equals

$$(-1)^{a_1} \sum_{s \in \Omega^{P_1}(\mathfrak{a}_P)} \sum_{\delta \in s^P \backslash P_1} B^s_{P|P_1}(s^{-1}(H(\delta x)-T))(M(s, \lambda)\psi)(\delta x, s\lambda+\rho_s) .$$

12.2. Computation of an inner product.

Let Q be an ε-invariant parabolic subgroup and let $P \subset P_1 \subset Q$. We consider functions ψ and θ cuspidal on P attached to unitary representations π and χ with central characters ω_π and ω_χ trivial on $A_P(\mathbf{R})^o$.

We are looking for a rather explicit formula for the function $\omega^\varepsilon_{P_1}(x, \lambda, \mu, \psi, \theta)$ defined to be the integral over $P_1 \backslash \mathbf{P}^1_1$ of

$$p \longmapsto \Lambda^{T,P_1} E_{P_1}(px, \psi, \lambda) E_Q(\varepsilon(px), \theta, \mu) .$$

We shall assume that $\mathrm{Re}(\lambda)$ is sufficiently regular and in particular Lemma 12.1.2 applies. We obtain a sum over $s \in \Omega^{P_1}(\mathfrak{a}_P)$ of an integral over $P_1 \backslash \mathbf{P}^1_1$ of a sum over $sP \backslash P_1$ and we may combine the sum and the integral if the resulting expression is absolutely convergent.

This amounts to proving that the integral over $sP \backslash \mathbf{P}^1_1$ of the absolute value of

$$p \longrightarrow B^s_{P|P_1}(\hat{s}(H(px)-T)M(s, \lambda)\psi(px, s\lambda+\rho_s)E_Q(\varepsilon(px), \theta, \mu)$$

is finite. Since the Eisenstein series E_Q is known to be slowly increasing, that is, that

$$|x|^{-N}|E_Q(x,\ \theta,\ \mu)|$$

is bounded for some N, we need only to show that given N',

$$|am|^{N'}|B^s_{P|P_1}(s^{-1}(H(a)-T))a^{s\lambda+\rho_s}||\psi(mx,\ s\lambda+\rho_s|$$

Should be $M(s,\lambda)\psi$ and m should lie in ${}_sM$.

is bounded for $a \in A^{P_1}_{sP}(\mathbf{R})^o$ and m in a Siegel domain of $\mathbf{M}^1$, uniformly for x in a compact set provided that $\mathrm{Re}(\lambda)$ is sufficiently regular. We have assumed that ψ is cuspidal on P and hence

$$|m|^{N'}|\psi(mx,\ s\lambda+\rho_s|$$

is bounded when m is in a Siegel set of $\mathbf{M}^1$ and x in a compact set. The boundedness of

$$|a|^{N'}|B^s_{P|P_1}(s^{-1}(H(a)-T))a^{s\lambda+\rho_s}|$$

$s^{-1}H \in {}^-\mathfrak{a}$

$H \in s({}^-\mathfrak{a})$

$s\lambda(sX) = \lambda(X)$

is an immediate consequence of the following

LEMMA 12.2.1. <u>The support of</u> $B^s_{P|P_1}$ <u>is contained in the cone</u> $\varpi(X) \leq 0$ <u>for all</u> $\varpi \in \hat{\Delta}^{P_1}_{sP}$. ; think the s belongs here! No!

The proof of this property has already been given in the proof of Lemma 9.2.7. We repeat the argument. Up to a sign $B^s_{P|P_1}$ is the characteristic function of the set of $X \in \mathfrak{a}^{P_1}_{sP}$ such that (i) $\varpi_\alpha(sX) > 0$ if $\alpha \in \Delta^{P_1}_{sP} - \Delta^{P_1 \cap P_s}_{sP}$, i.e., if $s^{-1}\alpha < 0$ and (ii) $\varpi_\alpha(sX) \leq 0$ if $\alpha \in \Delta^{P_1 \cap P_s}_{sP}$, i.e., if $s^{-1}\alpha > 0$; and hence

(i) comes from support of all $\hat{\tau}^{P_1}_R(sX)$

(ii) comes from alternation.

SIGN: comes from terms with no alternation i.e. $R = P_1 \cap P_s$

~~$\phi^s_{P|P_1}$ seems to be essentially the char. fn of $-\mathfrak{a}^{P_1}_P$~~

$$(X, \Lambda) = \sum_{\alpha \in \Delta_{sP}^{P_1}} \varpi_\alpha(sX).s^{-1}\alpha(\Lambda) \leq 0$$

for any regular $\Lambda \in \mathfrak{a}_{sP}^{P_1 +}$. □

Then provided that $\mathrm{Re}(\lambda)$ is sufficiently regular we see that $\omega^\varepsilon_{P_1}(x, \lambda, \mu, \psi, \theta)$ is the sum over $s \in \Omega^{P_1}(\mathfrak{a}_P)$ of the integral over $s\mathbf{N}sP\backslash \mathbf{P}^1_1 = sM\backslash s\mathbf{M}^1 \times \mathbf{A}^{P_1}_{sP} \times (K \cap \mathbf{P}_1)$ of the product of

$$(-1)^{a_1} B^s_{P|P_1}(s^{-1}(H(px)-T))M(s, \lambda)\psi(px, s\lambda+\rho_s)a(p)^{-2\rho_s}$$

times

$$\overline{E^{\varepsilon sP}_Q(\varepsilon(px), \theta, \mu)} \quad .$$

It may be convenient to transform the last term. Let $\theta_\varepsilon(x) = \theta(\varepsilon(x))$ and $\mu_\varepsilon(H) = \mu(\varepsilon(H))$, then this term equals

$$\overline{E^{sP}_Q(px, \theta_\varepsilon, \mu_\varepsilon)}$$

which in turn equals the sum over $t \in \Omega^Q(\varepsilon^{-1}\mathfrak{a}_P, s\mathfrak{a}_P)$ of

$$\overline{M(t, \mu_\varepsilon)\theta_\varepsilon(px, t\mu_\varepsilon+\rho_s)} \quad .$$

Summing up we get the

LEMMA 12.2.2. <u>Assume that</u> ψ <u>and</u> θ <u>are cuspidal on</u> P <u>then the following equality of meromorphic function holds for</u> x <u>in</u> $A_1(\mathbb{R})^0K$:

$$\omega^{\varepsilon}_{P_1}(x,\ \lambda,\ \mu,\ \psi,\ \theta) = \sum_{s \in \Omega^{P_1}(\mathfrak{a}_P)} \ \sum_{t \in \Omega^{Q}(\varepsilon^{-1}\mathfrak{a}_P, s\mathfrak{a}_P)} a(x)^{s\lambda + t\bar{\mu}_\varepsilon}$$

$$(-1)^{a_1} \hat{B}^{s}_{P|P_1}(s^{-1}T,\ (\lambda + s^{-1}t\bar{\mu}_\varepsilon))(M(s,\ \lambda)\psi^{x},\ M(t,\ \mu_\varepsilon)\theta^{x}_{\varepsilon})_{sM^1}$$

where $\psi^{x}(p) = \psi(px)$, _the scalar product_ $(\ ,\ )_{sM^1}$ _is the scalar product in_ $L^2(sM \backslash sM^1)$ _and_

$$\hat{B}^{s}_{P|P_1}(x,\ \lambda) = \int_{\mathfrak{a}^{P_1}_{P}} B^{s}_{P|P_1}(H - X)e^{\lambda(H)}\,dH \quad .$$

The two members are well defined and equal when $Re(\lambda)$ and $Re(\mu)$ are sufficiently regular; they have meromorphic analytic continuation in $(\lambda,\ \bar{\mu})$ to the whole space and hence are equal everywhere. □

12.3. _Some application_.

According to Lectures 10 and 11, the "right-hand side" of the trace formula is the sum over pairs of parabolic subgroups $P_1 \subset P_2$ such that there exists one and only one ε-invariant parabolic subgroup Q between P_1 and P_2 of terms $J^{P_2}_{P_1}$ which are equal to the sum over $w \in P_1 \backslash Q/\varepsilon(P_1)$ of the integral over $P_1 \cap \varepsilon^{-1}w^{-1}(P_1) \backslash \mathbb{G}'_{\varepsilon}$ of

$${}_{\varepsilon}\sigma^{2}_{1}(H(x) - T)K_{P_1}(x,\ w\varepsilon(x)) \quad .$$

Using the spectral decomposition it was shown that $K_{P_1}(x,\ y)$ is equal to the sum over $P \subset P_1$ and over $\pi \in \prod_2(M_P)$ of not defined but meaning fairly obvious

$$\frac{1}{n_1(A_P)} \int_{i(\mathfrak{a}_P^{P_1})^*} \Xi_\pi^{P_1}(x, y, \lambda, \lambda)\, d\lambda$$

where $\Xi_\pi^{P_1}(x, y, \lambda, \nu)$ is given by

$$\int_{i\mathfrak{a}_{P_1}^* - \Lambda} \sum_{\psi \in B_\pi} \Lambda^{T_1} {}^{P_1}E_{P_1}(x, I_{\lambda+\lambda_1}(\phi)\psi, \lambda+\lambda_1)$$

$$\overline{E_{P_1}(y, \psi, -\bar{\nu}-\bar{\lambda}_1)}\, d\lambda_1 \ .$$

In this expression λ and ν belong to $(\mathfrak{a}_P^{P_1})^* \otimes \mathbf{C}$ and $\Lambda \in \mathfrak{a}_{P_1}$. We assume ϕ to be K-finite and the sum over $\psi \in B_\pi$ may be assumed to reduce to a finite sum. We have not included Λ in the notation $\Xi_\pi^{P_1}$ since it is in fact independent of Λ. To see this one should remark that $\lambda_1 \longrightarrow I_{\lambda+\lambda_1}(\phi)\psi$ is of Paley-Wiener type since ϕ is compactly supported and hence we are free to shift the integration domain. As a function of (λ, ν) it is meromorphic and we have tacitly assumed that (λ, ν) is not a singular value.

The main result of Lecture 11 may be stated in the following way: the sum over π and w and the integral over $i(\mathfrak{a}_P^{P_1})^*$ and $P_1 \cap \varepsilon^{-1}w^{-1}(P_1)\backslash \mathbf{G}_\varepsilon^1$ of $\Xi_\pi^{P_1}(x, w\varepsilon(x), \lambda, \lambda)$ is absolutely convergent, so that we are free to interchange the order of those sums and integrals. Before using this we need some preparation.

LEMMA 12.3.1. <u>The series</u>

$$\sum_{w \in P_1\backslash Q/\varepsilon(P_1)} \ \sum_{\xi \in P_1 \cap \varepsilon^{-1}w^{-1}(P_1)\backslash P_1} \Xi_\pi^{P_1}(x, w\varepsilon(\xi x), \lambda, \nu)$$

<u>is absolutely convergent and defines a meromorphic function of</u> (λ, ν) <u>which we shall denote by</u> $Z_\pi^{P_1}(x, \lambda, \nu)$.

We first remark that the series may be written

$$\sum_{\xi \in P_1 \backslash Q} \Xi_\pi^{P_1}(x, \xi\varepsilon(x), \lambda, \nu) \quad .$$

Now consider $a_1 \in A_1(\mathbb{R})^o$ and $y \in \mathbb{N}_1 \mathbb{M}_1^1 K$ then

$$E_{P_1}(a_1 y, \psi, -\overline{\nu}-\overline{\lambda}_1) = a_1^{-\overline{\lambda}_1} E_{P_1}(y, \psi, -\overline{\nu}) \quad .$$

Recall that

$$\lambda_1 \longrightarrow \Lambda^{T,P_1} E_{P_1}(x, I_{\lambda+\lambda_1}(\phi)\psi, \lambda+\lambda_1)$$

is of Paley-Wiener type on $i\mathfrak{a}_1^* \otimes \mathbf{C}$; this implies that

$$a_1 \longrightarrow \Xi_\pi^{P_1}(x, a_1 y, \lambda, \nu)$$

is compactly supported in some compact set $\omega \subset A_1(\mathbb{R})^o$ independent of y. From this we deduce that the series reduces to a finite sum uniformly when x, λ, ν are in compact set in the holomorphy domain for (λ , ν). □

LEMMA 12.3.2. <u>Assume that</u> $\mathrm{Re}(-\nu) + \Lambda$ <u>is sufficiently regular in</u> $\mathfrak{a}_Q^*$ <u>then</u> $Z_\pi^{P_1}(x, \lambda, \nu)$ <u>equals</u>

$$\int_{i\mathfrak{a}_{P_1}^* - \Lambda} \Lambda^{T,P_1} E_{P_1}(x, I_{\lambda+\lambda_1}(\phi), \lambda+\lambda_1) \overline{E_Q(\varepsilon(x), \psi, -\overline{\nu}-\overline{\lambda}_1)}\, d\lambda_1 \quad .$$

This is an immediate consequence of the fact that when $\mathrm{Re}(-\nu-\lambda_1)$ is sufficiently regular, then

$$E_Q(y,\ \psi,\ -\overline{\nu}-\overline{\lambda}_1) = \sum_{\xi \in P_1\backslash Q} E_{P_1}(\xi y,\ \psi,\ -\overline{\nu}-\overline{\lambda}_1)\ . \quad \square$$

LEMMA 12.3.3. <u>The function</u> $p \longrightarrow Z_\pi^{P_1}(px,\ \lambda,\ \nu)$ <u>is integrable over</u> $P_1\backslash \mathbf{P}_1^1$ <u>and its integral defines a meromorphic function</u> $S_\pi^{P_1}(x,\ \lambda,\ \nu)$.

Lemma 6.6 shows that

$$p \longrightarrow \Lambda^{T_1 P_1} E_{P_1}(px,\ I_{\lambda+\lambda_1}(\phi)\psi,\ \lambda+\lambda_1)$$

is a "rapidly decreasing" function in a Siegel domain $\mathfrak{S}$ in $\mathbf{P}_1^1$, uniformly in λ_1 since λ_1 is trivial on $\mathbf{P}_1^1$, and hence all we need to prove is that

$$p \longrightarrow \sum_{\substack{\xi \in P_1\backslash Q \\ a_1(\xi\varepsilon(px)) \in \omega}} E_{P_1}(\xi\varepsilon(px),\ \psi,\ -\overline{\nu})$$

is slowly increasing in $\mathfrak{S}$. To see this we consider $w \in P_1\backslash Q/P_0$ and N_0^w a subgroup of N_0 isomorphic to $N_0 \cap w^{-1}P_1 w\backslash N_0$; using the slow increase property of Eisenstein series, that is, that

$$|E(y,\ \psi,\ -\overline{\nu})| \leq c|y|^N$$

for some N and some constant c, all we need to remark is that given a compact set $\omega' \subset A_1(\mathbf{R})^0$

$$a \longrightarrow \sum_{\substack{\eta \in N_0^w \\ a_1(w\eta a) \in \omega'}} |\eta|^N$$

is "slowly increasing" for $a \in A_0(\mathbb{R})^o$. All those evaluations are uniform for x, λ, ν in compact sets outside the singular set. ☐

The main result of Lecture 11 may be restated using the functions $S_\pi^{P_1}$: the $J_{P_1}^{P_2}$ are equal to the sum over $P \subset P_1$ and over $\pi \in \prod_2(M_P)$ of

$$\frac{1}{n_{\mathfrak{S}_1}(A_P)} \int_{i(\mathfrak{a}_P^{P_1})^*} \int_{\mathbf{P}_1^1 \backslash \mathbf{G}'_\varepsilon} {}_\varepsilon\sigma_1^2(H(x)-T) S_\pi^{P_1}(x, \lambda, \lambda)\, d\lambda dx \ .$$

Using Lemma 12.2.2 a more concrete formula for $S_\pi^{P_1}(x, \lambda, \nu)$ can be given for some π and some values of λ and ν; this is the aim of the next

LEMMA 12.3.4. <u>Assume that</u> $Re(\lambda)$ <u>and</u> $Re(-\nu+\Lambda)$ <u>are suitably regular, then if</u> $x \in A_1(\mathbb{R})^o K$ <u>and</u> π <u>is cuspidal on</u> M:

$$S_\pi^{P_1}(x, \lambda, \nu) = \sum_{\substack{s \in \Omega^{P_1}(\mathfrak{a}_P) \\ t \in \Omega_\varepsilon^Q(\mathfrak{a}_P, \mathfrak{a}_P)}} \int_{i\mathfrak{a}_{P_1}^* - \Lambda} d\lambda_1$$

$\subseteq \Omega^Q(\quad) \rtimes E$: $\mathfrak{a}_P \to \mathfrak{a}_P$: $\to \varepsilon^{-1}$: minimal length

$$a(x)^{\lambda_1 - t(\nu+\lambda_1)} \hat{B}^s_{P|P_1}(s^{-1}T, \lambda - t(\nu+\lambda_1))$$

$+ -t(\bar{\nu} + \bar{\lambda}_1)$?

$$\sum_{\psi \in B_\pi} (M(s, \lambda) I_{\lambda+\lambda_1}(\phi)\psi^x, M(st\varepsilon, -\bar{\nu}-\bar{\lambda}_1)\psi_\varepsilon^x) \ .$$

Can replacing $\lambda+\lambda_1$ $\quad (\lambda+\lambda_1) - t(\nu+\lambda_1)$

Under the regularity assumptions all integrals and series are absolutely convergent and we may appeal to Lemma 12.2.2 since π is cuspidal. □