$w_{tt} = w_{xx} + 0$; $w(0,t) = 0 = w(1,t)$

$w(x,0) = u(x,0) - v(x)$; $w_t(x,0) = u_t(x,0) = 0$

With RHS $= 0$, boundary conditions $= 0$, we use separation of variables.

$w = XT$.

$\frac{T''}{T} = \frac{X''}{X} = -\lambda^2$; $X(0) = 0 = X(1)$

$\Rightarrow X_n = \sin(n\pi x)$; $\alpha_n = \int_0^1 \sin^2(n\pi x)\,dx = \int_0^1 \frac{1-\cos(2n\pi x)}{2}\,dx$

$\lambda_n = n\pi$

$= \frac{x}{2} - \frac{1}{4n\pi}\sin(2n\pi x)\Big|_0^1$

$= \frac{1}{2}$

$\varphi_n(x) = \sqrt{2}\sin(n\pi x)$.

$T_n'' + (n\pi)^2 T_n = 0 \Rightarrow T_n(t) = a_n\cos(n\pi t) + b_n\sin(n\pi t)$.

$\Rightarrow w(x,t) = \sum_{n=1}^{\infty} \{a_n\cos(n\pi t) + b_n\sin(n\pi t)\}\,\varphi_n(x)$.

$w(x,0) = f(x) - v(x) = \sum_{n=1}^{\infty} a_n\varphi_n(x) \Rightarrow a_n = \int_0^1 [f(x) - v(x)]\,\varphi_n(x)\,dx$.

$w_t(x,0) = 0 = \sum_{n=1}^{\infty} b_n \cdot n\pi\,\varphi_n(x) \Rightarrow b_n \cdot n\pi = 0 \Rightarrow b_n = 0$.

$w(x,t) = \sum_{n=1}^{\infty} a_n\cos(n\pi t)\,\varphi_n(x)$

III. Solve the initial-value problem:

$$\begin{cases} u_t = u_{xx} - u_x + te^{-x}, & 0<x<1,\ 0<t \\ u(0,t) = 0 = u(1,t); & u(x,0) = f(x). \end{cases}$$