Inner Products **Name:**
Worksheet for Math 223
Patrick Brosnan, Instructor
Please take home and return by April 6.

Reading List. Here is a list of readings from FIS from material that we will have covered in class up from the first class.

1. Appendices A-C.
2. Chapter 1.1-6.
3. Chapter 2.1-4.
4. Section 3.2-4.
5. Chapter 4.1-3.
6. Chapter 5.1-3.
7. Chapter 6.1-2

Practice Problems. Do the following problems from FIS for practice but do not turn them in.
6.1 1
6.2 1, 2

Problem 1. Let $x = (2, 3, i)$, $y = (1 + i, 2, 7)$ be two vectors in $\mathbf{C}^3$ with the standard inner product. Compute $\langle x, y\rangle$, $\|x\|$, $\|y\|$ and $\|x + y\|$. Then verify the Cauchy-Schwartz inequality.

Problem 2. Let $C([0, 1])$ denote the set of continous real-valued functions on the closed inteval $[0, 1]$ with the inner product $\langle f, g\rangle = \int_0^1 fg\,dx$. Let V denote the subspace spanned by 1 and e^x. Find an orthonormal basis for V.

Problem 3. Suppose T is a linear transformation on a finite dimensional vector space V such that $|T(v)| = |v|$ for all $v \in V$. Show that T is an isomorphism. (Hint: first show that T is one-to-one.)

Problem 4. Find an orthogonal basis for the subspace V of $\mathbf{R}^4$ spanned by $(1, 1, 1, 1)$, $(1, 2, 3, 4)$ and $(1, 2, 1, 2)$.

Problem 5. Find an orthogonal basis for the intersection of the subspace V of $\mathbf{R}^4$ given in problem 4 with the subspace W spanned by $(1, 0, 0, 0), (0, 1, 0, 0)$ and $(0, 0, 1, 1)$. (**Hint:** A vector v in $\mathbf{R}^4$ is in W if and only if the component of v perpendicular to w is 0.)