

Math 120  
Problem Set 6  
Due Friday, November 28  
Patrick Brosnan, Instructor

*Reading.* Reread Chapters 1-4 to study for exam.

*Practice.* Do the following exercises in the book but do not turn them in for a grade.

- (4.2) 19, 25, 33, 41
- (4.3) 14, 31, 33
- (4.4) 33
- (4.5) 1-23 odd, 37
- (4.8) 1-21 odd
- (4.9) 1-23 odd

*Mandatory assignment.* Turn in solutions to the following problems from the book.

- (4.1) 31, 35
- (4.2) 47
- (4.3) 37, 38, 41
- (4.5) 38
- (4.8) 31, 34, 35
- (4.9) 27, 35

In addition, turn in a solution to the following problem.

1. In class, we proved the IVT and EVT using the least upper bound property of the real numbers as an axiom. However, this can actually be reversed. That is, as we will prove in this exercise, if we assume the IVT (instead of the least upper bound property) as an axiom, then the least upper bound property of the real numbers is a consequence.

Suppose then that the IVT holds and let  $S$  be a non-empty subset of  $\mathbb{R}$  which is bounded from above. To get a contradiction, suppose that  $S$  does not have a least upper bound. Define a function  $h : \mathbb{R} \rightarrow \mathbb{R}$  by

$$h(x) = \begin{cases} 1, & \text{if } x \text{ is an upper bound for } S; \\ -1, & \text{otherwise.} \end{cases}$$

- (a) Show that  $h$  takes both the value 1 and the value  $-1$ .
- (b) Show that  $h$  is continuous on  $\mathbb{R}$ .
- (c) Use the IVT to get a contradiction.