1. Let $M(x) = \sum_{n \leq x} \mu(n)$. Suppose that $M(x) = O(x^\theta)$ for some $\theta < 1$ and all $x \geq 1$. Prove that
\[
\frac{1}{\zeta(s)} = s \int_{1}^{\infty} \frac{M(x)}{x^{s+1}} \, dx
\]
for $\Re(s) > \theta$, and hence that $\zeta(s) \neq 0$ for $\Re(s) > \theta$.

2. Show that the smallest prime $p$ that does not divide the integer $n$ satisfies $p \leq \log n (1 + o(1))$.

3. Let $H_n = \sum_{k=1}^{n} \frac{1}{k}$. Write $H_n = \frac{a_n}{b_n}$ where $a_n, b_n$ are positive integers with $\gcd(a_n, b_n) = 1$.

   Show that $\log a_n = n + o(n)$ and $\log b_n = n + o(n)$.