1. Let \( \mu(n) \) be the Möbius function. Show that
\[
\sum_{n=1}^{\infty} \frac{\mu(n)}{n^2} = \frac{6}{\pi^2}.
\]

2. Let \( d(n) \) be the number of divisors of \( n \). Prove that the following identity holds for \( \sigma > 1 \) (where as usual \( s = \sigma + it \)):
\[
\sum_{n=1}^{\infty} \frac{d(n^2)}{n^s} = \frac{\zeta^3(s)}{\zeta(2s)}.
\]

3. Let \( \nu(n) \) denote the number of distinct prime divisors of \( n \) so \( \nu(1) = 0 \) and \( \nu(p_1^{a_1} \cdots p_k^{a_k}) = k \). Show that the following holds for \( \sigma > 1 \):
\[
\sum_{n=1}^{\infty} \frac{\nu(n)}{n^s} = \zeta(s) \sum_p \frac{1}{p^s}.
\]

4. (a) Prove that the series \( \sum_{n=1}^{\infty} n^{-1-it} \) has bounded partial sums if \( t \neq 0 \), but unbounded partial sums if \( t = 0 \).
(b) Prove that the series \( \sum_{n=1}^{\infty} n^{-1-it} \) diverges for all real \( t \).

5. Prove that
\[
\sum_{\substack{m \geq 1 \\ n \geq 1 \\ (m,n)=1}} \frac{1}{m^2n^2} = \frac{\zeta^2(2)}{\zeta(4)} = \frac{5}{2}.
\]
(You may assume as known the explicit values for \( \zeta(2) \) and \( \zeta(4) \).)

6. Let \( \phi \) be the Euler phi function and let \( F(s) = \sum_{m=1}^{\infty} \phi(m)^{-s} \).
(a) Show that the series converges uniformly in compact subsets of the half plane \( \sigma > 1 \).
(b) Show that
\[
F(s) = \prod_p \left( (1 + (p-1)^{-s}(1 - p^{-s})^{-1}) = \zeta(s)Q(s),
\]
where
\[
Q(s) = \prod_p (1 - p^{-s} + (p-1)^{-s}),
\]
and that this product converges absolutely and uniformly in compact subsets of the half-plane \( \sigma > 0 \).
(c) Define \( a(n) = \# \{m : \phi(m) = n \} \) so that \( F(s) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} \). Is \( a(n) \) multiplicative? Is the product in (b) an Euler product for \( F(s) \)?