Practice Problems for MATH 226

1. Find and classify as local maximum, local minimum, or saddle point the critical points of

$$f(x,y) = x^3 - 2xy + \frac{y^2}{2}.$$

2. Calculate

$$\lim_{(x,y)\to(0,0)} \frac{3xy}{\sqrt{x^2+y^2}}.$$

3. If $z = e^x \tan y$, where $x = s^2 + t^2$ and $y = st$, find $\frac{\partial z}{\partial t}$ when $s = 1$ and $t = 0$.

4. Let $P = (1,2,3)$, $Q = (1,-1,-2)$ and $R = (0,0,0)$. Find an equation of the plane through P, Q and R. Then find the area of the triangle formed by PQR. Finally, find the equation of the line through P, perpendicular to the plane through P, Q and R.

5. Given $f(x,y) = x^2 - 5xy$, find $\nabla f(x,y)$, the directional derivative at $(2,1)$ in the direction of $\overline{u} = -\overline{i} + 3\overline{j}$, and the linearization of f at $(2,1)$. Then use the linearization to approximation $f(1.9, 0.9)$.

6. Evaluate $\int\int\int_D (x^2+y^2+z^2)dV$, where D is the solid lying inside the sphere of radius 1 centred at $(0,0,1)$, and inside (i.e. above) the cone $x^2 + y^2 = 3z^2$.

7. A wire 12 cm long is cut into three or fewer pieces, with each piece bent into a square. What is the minimal total area of the squares? What is the maximal total area of the squares?

8. Evaluate

$$\int\int_R \sin\left(\frac{1}{2}(x+y)\right)\cos\left(\frac{1}{2}(x-y)\right) dA,$$

where R is the triangular region whose vertices are $(0,0), (2,0)$ and $(1,1)$.

9. Use Lagrange multipliers to find the maximum and minimum values of the function $2x^2 + 4xy - y^2$ on the circle. $x^2 + y^2 = 1$.