

Practice Midterm 2 for MATH 226

1. The radius r and height h of a right circular cylinder are measured with possible errors of 4% and 5%, respectively. Approximate the maximum possible error percentage in measuring the volume of the cylinder. The percentage error is the ratio of the amount of error to the original amount, in this case $\Delta V/V$.

2. The x and y components of a fluid moving in two dimensions are given by the following functions :

$$u(x, y) = 2y \quad \text{and} \quad v(x, y) = -2x, \quad x \geq 0, \quad y \geq 0.$$

The speed of the fluid at the point (x, y) is given by

$$S(x, y) = \sqrt{u(x, y)^2 + v(x, y)^2}.$$

Find $\frac{\partial S}{\partial x}$ and $\frac{\partial S}{\partial y}$.

3. Define $g(x, y)$ by

$$g(x, y) = \begin{cases} (x^2 + y^2) \sin\left(\frac{1}{\sqrt{x^2 + y^2}}\right) & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

Show that $g(x, y)$ is differentiable at $(x, y) = (0, 0)$. Is $\frac{\partial g}{\partial x}$ continuous at $(x, y) = (0, 0)$?

4. Let $h(x, y)$ be defined by

$$h(x, y) = xf(x + y) + yg(x + y),$$

where f and g are functions of a single variable with continuous derivatives. Show that the second partials of h ,

$$\frac{\partial^2 h}{\partial x^2}, \quad \frac{\partial^2 h}{\partial x \partial y} \quad \text{and} \quad \frac{\partial^2 h}{\partial y^2}$$

are **linearly dependent**. That is, show that there exist constants a, b and c , not all zero, such that

$$a \frac{\partial^2 h}{\partial x^2} + b \frac{\partial^2 h}{\partial x \partial y} + c \frac{\partial^2 h}{\partial y^2} = 0.$$

5. For what values of x and y does the surface

$$z = 3xy - x^3 - 3y^4$$

have a horizontal tangent plane?