The University of British Columbia

October 5th, 2022

Practice Midterm for MATH 226

Closed book examination Time: 50 minutes

Last Name _____ **First** _____

Signature _____

Student Number _____

Special Instructions:

No memory aids are allowed. No calculators. No communication or other electronic devices. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. **Where boxes are provided for answers, put your final answers in them.**

Midterms written in pencil will not be considered for regrading.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		10
2		15
3		15
4		10
Total		50

1. Determine whether each of the sets below is open, closed or neither. What is the boundary and interior of each set?

(a) $\{(x, y, z) \in \mathbb{R}^3 \ : \ z > \sqrt{x^2 + y^2},\ x^2 + y^2 + z^2 \leq 2\}$

(b) $\{(x, y, z) \in \mathbb{R}^3 \ : \ x > 0,\ 0 \leq y \leq 3\}$

2. The points P, Q and R in $\mathbb{R}^3$ have coordinates $P = (1, 1, -1)$, $Q = (2, -1, 0)$ and $R = (2, 3, 3)$.

(a) Find the area of the triangle ΔPQR.

(b) Find the angle at P in the triangle ΔPQR. An answer in the form $\cos^{-1}(\cdot)$ or $\sin^{-1}(\cdot)$ is sufficient.

(c) Find the equation (in the form $Ax + By + Cz = D$) of the plane through the points P, Q and R.

3. (a) Prove that the line $x = t + 1, y = 3t, z = t + 3$ is parallel to the plane $4x - y - z = 2$.

(b) Find the equation (in the form $Ax + By + Cz = D$) of the plane that contains the line $x = t + 1, y = 3t, z = t + 3$ and is perpendicular to the plane $4x - y - z = 2$.

(c) Let $P = (2, 5, 1)$. Find the point Q on the line $x = t + 1, y = 3t, z = t + 3$ that is closest to P.

4. (a) Suppose that $\overline{v}$ is a vector in $\mathbb{R}^3$ with the property that $\overline{v} \times \overline{w} = \overline{0}$, the zero vector, for **every** vector $\overline{w}$ in $\mathbb{R}^3$. What can you conclude about $\overline{v}$? Prove your answer.

(b) Suppose that $\overline{a}$ is a nonzero vector in $\mathbb{R}^3$, that $\overline{b}$ is a vector in $\mathbb{R}^3$, and that the vector equation $\overline{a} \times \overline{x} = \overline{b}$ has at least one solution in vectors $\overline{x}$ in $\mathbb{R}^3$. Show that $\overline{a} \cdot \overline{b} = 0$.