**Math 226, 2022 : Solutions for Assignment 1**

**Section 10.1, Question 10**

The distance to the origin of the vector $(1, 1, \ldots, 1)$ in $\mathbb{R}^n$ is

$$\sqrt{1^2 + 1^2 + \cdots + 1^2} = \sqrt{n}.$$

**Section 10.1, Question 22**

$z \geq \sqrt{x^2 + y^2}$ represents every point whose distance above the xy-plane is not less than its horizontal distance from the z-axis. It therefore consists of all points inside and on a circular cone with axis along the positive z-axis, vertex at the origin, and semi-vertical angle 45 degrees.

**Section 10.1, Question 31**

The points in $\mathbb{R}^3$ that satisfy $x^2 + y^2 \leq 1$ and $z \geq y$ are those that are inside or on the vertical. cylinder $x^2 + y^2 = 1$, and are additionally above or on the plane $z = y$.

**Section 10.1, Question 36**

If $S = \{(x, y)\ :\ |x| + |y| \leq 1\}$, then the boundary of $S$ is the set of points on the edge of the square with vertices $(\pm 1, 0)$ and $(0, \pm 1)$. The interior of $S$ is all the points inside this square. $S$ is closed since it contains all its boundary points. It is also bounded since every point in it is at most distance 1 from the origin.

**Section 10.2, Question 9**

Choose the vector $\bar{i}$ to point East and $\bar{j}$ to point North, and write the wind velocity as

$$\overline{v}_{\text{wind}} = a\bar{i} + b\bar{j}.$$

Then

$$\overline{v}_{\text{wind}} = \overline{v}_{\text{wind rel car}} + \overline{v}_{\text{car}}$$

When $\overline{v}_{\text{car}} = 50\bar{j}$, the wind appears to come from the west, so that

$$\overline{v}_{\text{wind rel car}} = \kappa\bar{i},$$

for some real number $\kappa$. We thus have

$$a\bar{i} + b\bar{j} = \kappa\bar{i} + 50\bar{j},$$

so that $a = \kappa$ and $b = 50$. Since we also know that when $\overline{v}_{\text{car}} = 100\bar{j}$, the wind appears to come from the the northwest, it follows that

$$\overline{v}_{\text{wind rel car}} = \omega\left(\bar{i} - \bar{j}\right),$$

for some real number $\omega$. We thus have

$$a\bar{i} + b\bar{j} = \omega\left(\bar{i} - \bar{j}\right) + 100\bar{j},$$

whence $a = \omega$ and $b = 100 - \omega$. It follows that

$$\omega = a = b = 50,$$

whereby we can conclude that the wind is from the southwest at $50\sqrt{2}$ km/hr.

**Section 10.2, Question 21**

If we have

$$\overline{r} \cdot \overline{a} = b,$$

then the vector projection of $\overline{r}$ along $\overline{a}$ is given by

$$\frac{\overline{r} \cdot \overline{a}}{|\overline{a}|} \frac{\overline{a}}{|\overline{a}|} = \frac{b}{|\overline{a}|^2}\overline{a}.$$

We conclude that the equation $\overline{r} \cdot \overline{a} = b$ is satisfied by all points on the plane through the vector $\frac{b}{|\overline{a}|^2}\overline{a}$, that is normal to $\overline{a}$.

**Section 10.2, Question 25**

A unit vector in the direction of $\overline{u}$ is given by $\frac{\overline{u}}{|\overline{u}|}$, while a unit vector in the direction of $\overline{v}$ is given by $\frac{\overline{v}}{|\overline{v}|}$. It follows that $\frac{\overline{u}}{|\overline{u}|} + \frac{\overline{v}}{|\overline{v}|}$ bisects the angle between $\overline{u}$ and $\overline{v}$. To find a unit vector with this property, normalize to get the vector

$$\frac{\frac{\overline{u}}{|\overline{u}|} + \frac{\overline{v}}{|\overline{v}|}}{\left|\frac{\overline{u}}{|\overline{u}|} + \frac{\overline{v}}{|\overline{v}|}\right|}.$$

**Section 10.2, Question 33**

Since the vector $\overline{a}$ satisfies $|\overline{a}|^2 > 4rst$, there exists a positive real number $\kappa$ such that we may write

$$|\overline{a}|^2 = 4rst + \kappa^2.$$

Since $r\overline{x} + s\overline{y} = \overline{a}$, we have

$$|\overline{a}|^2 = \overline{a} \cdot \overline{a} = (r\overline{x} + s\overline{y}) \cdot (r\overline{x} + s\overline{y}) = r^2|\overline{x}|^2 + s^2|\overline{y}|^2 + 2rs\overline{x} \cdot \overline{y}.$$

From the fact that $\overline{x} \cdot \overline{y} = t$, it follows that

$$\kappa^2 = r^2|\overline{x}|^2 + s^2|\overline{y}|^2 - 2rs\overline{x} \cdot \overline{y} = |r\overline{x} - s\overline{y}|^2$$

and so $\kappa = |r\overline{x} - s\overline{y}|$. We thus can write $r\overline{x} - s\overline{y} = \kappa\overline{u}$, where $\overline{u}$ is a **unit** vector. We thus have the simultaneous equations

$$\begin{cases} r\overline{x} + s\overline{y} = \overline{a} \\ r\overline{x} - s\overline{y} = \kappa\overline{u}. \end{cases}$$

Add and subtract these and use the fact that $r \neq 0$ and $s \neq 0$ to conclude that

$$\overline{x} = \frac{\overline{a} + \kappa\overline{u}}{2r} \quad \text{and} \quad \overline{y} = \frac{\overline{a} - \kappa\overline{u}}{2s}.$$

Here, again, $\kappa = \sqrt{|\overline{a}|^2 - 4rst}$ and $\overline{u}$ is (any) unit vector.