All questions come from the course text. Problems 1–3 cover related rates, problems 4–6 cover optimization and extrema, and problems 7–9 cover second derivatives and applications.

1. *(Section 3.11, exercise 5.)* The sides of a square increase in length at a rate of $2m/s$.

   a. At what rate is the area of the square changing when the sides are $10m$ long?
   b. At what rate is the area of the square changing when the sides are $20m$ long?
   c. Draw a graph that shows how the rate of change of the area varies with the side length.

**Solution:** Let $s_0$ be the side length of the square at the time we begin measuring; then the side length at time $t \geq 0$ from when we start measuring is $s(t) = s_0 + 2t$ metres.

a. The area of a square of side length $s$ is $A(s) = s^2$. So, if the side length of our square is given by $s(t) = s_0 + 2t$, then its area at time $t$ is given by

   $$A(t) = A(s(t)) = (s_0 + 2t)^2 = s_0^2 + 4s_0t + 4t^2,$$

   and so

   $$A'(t) = 2(s_0 + 2t)(2) = 4s_0 + 8t = 4(s_0 + 2t) = 4s(t).$$

   In particular, when $s(t) = s_0 + 2t = 10$, we have that $A'(t) = 4(10) = 40m/s^2$.

b. Using our formula $A'(t) = 2s(t)$ from part a., we see that when $s(t) = 20$, $A'(t) = 4(20) = 80m/s^2$.

c. The question is asking for a graph of $A'(t)$; it is given below (with $s_0 = 0$).

![Graph of A'(t)](image-url)

We remark that the answers to these questions are unchanged by the value of $s_0$, so we could have assumed *without loss of generality* that $s_0 = 0$.

2. *(Section 3.11, exercise 19.)* A swimming pool is $50m$ long and $20m$ wide. Its depth decreases linearly along the length from $3m$ to $1m$ (from the deep end to the shallow end). It is initially empty and is filled in at a rate of $1m^3/min$. How fast is the water level rising 4 hours after the filling begins? How long will it take to fill the pool?
Solution: The shape of the swimming pool is that of a rectangular prism of dimensions $50m \times 20m \times 1m$ sitting atop a right triangular prism, the triangular faces of which have legs of lengths $2m$ by $50m$, which are $20m$ apart from each other (see below).

Clearly, we have to deal with the bottom (triangular) part of the pool separately from the top (rectangular) part.

When the pool first begins to fill, the water in the pool will collect in the bottom, resulting in a triangular prism (of water) whose triangular faces are similar triangles to those of the pool (see below, not to scale):

More precisely, if $b$ and $h$ are the quantities depicted in the above figure, then the ratio of $h$ to $b$ is always the same as the ratio of $2$ to $50$; that is,

$$\frac{h}{b} = \frac{2}{50} = \frac{1}{25} \iff b = 25h.$$  

Thus, until the bottom part of the pool is filled, the water in the pool has the shape of a right triangular prism whose triangular faces have legs of length $h$ and $25h$, which lie $20m$ apart, as $h$ varies between $0$ and $2$; the volume of such a prism is

$$V(h) = \frac{1}{2}(20m)(h m)(25h m) = 250h^2 \text{ m}^3,$$

and because we know that the amount of water in the pool after $t$ minutes of filling is $V(t) = t \text{ m}^3$, we have the equation

$$V(t) = 250h(t)^2 \iff h(t) = \sqrt{\frac{V(t)}{250}} = \frac{t^{1/2}}{250}.$$  

From this we deduce that

$$h'(t) = \frac{(1/2)t^{-1/2}}{25} = \frac{1}{50\sqrt{t}},$$
and so in particular that after 4 hours, i.e. 240 minutes, we have

\[ h'(240) = \frac{1}{50\sqrt{240}} = \frac{1}{200\sqrt{15}} \approx 0.001291 \text{ m/min.} \]

or about 1.29 mm/min.

The top part of the pool is easier to deal with. Each minute, 1 m\(^3\) of water fills the (rectangular) portion of the pool, creating a rectangular prism of volume 1 m\(^3\), length 50 m, and width 20 m; it follows that each minute water is pumped into the top portion of the tank, the level of water in the tank is rising at a rate of

\[ \frac{1 \text{ m}^3/\text{min.}}{(20 \text{ m})(50 \text{ m})} = \frac{1}{1000} \text{ m/min.}, \]

or one millimetre per minute.

The total volume of the top portion of the pool is (20 m)(50 m)(1 m) = 1000 m\(^3\), the same as the bottom portion; thus it will take 2(1000) = 2000 minutes, or 33 hours and 20 minutes, to fill the pool to the brim.

3. (Section 3.11, exercise 29.) An inverted conical water tank with a height of 12 feet and a radius of 6 feet is drained through a hole in the vertex at a rate of 2 cubic feet per second (see figure 1). What is the rate of change of the water depth when the water depth is 3 feet? (Hint: Use similar triangles.)

**Solution:** We refer to figure 1; as indicated by that diagram, if \( r(t) \) and \( h(t) \) denote respectively the base radius and height of the inverted water cone in the tank at time \( t \), then we have the constant ratio

\[ \frac{r(t)}{h(t)} = \frac{6 \text{ feet}}{12 \text{ feet}} = \frac{1}{2} \iff r(t) = \frac{h(t)}{2}. \]

The volume of the water cone is

\[ V(t) = \frac{1}{3} \pi r(t)^2 h(t) = \frac{1}{3} \pi \left(\frac{h(t)}{2}\right)^2 h(t) = \frac{\pi}{12} h(t)^3, \]

and so differentiating both sides of the equation, we obtain

\[ V'(t) = \frac{3\pi}{12} h(t)^2 h'(t) = \frac{\pi}{4} h(t)^2 h'(t). \]

It is given to us that \( V'(t) = -2 \) (negative because the volume of water in the tank is decreasing), and so we have the equation

\[ -2 = \frac{\pi}{4} h(t)^2 h'(t) \iff h'(t) = \frac{-8}{\pi h(t)^2}. \]

Therefore, in particular, when the depth of water is 3 feet, we will have that the water level in the tank will be dropping at a rate of

\[ \frac{8}{\pi(3)^2} = \frac{8}{9\pi} = 0.282932 \ldots \text{ feet per second}, \]

or about 3.4 inches per second.
4. (Section 4.1, exercises 19–21.) Sketch a graph of a function $f$ that is continuous on $[0, 4]$ and satisfies the given properties.

a. $f'(x) = 0$ when $x = 1$ and 2; $f$ has an absolute maximum at $x = 4$; $f$ has an absolute minimum at $x = 0$; $f$ has a local minimum at $x = 2$.

b. $f'(x) = 0$ when $x = 1, 2$, and 3; $f$ has an absolute minimum at $x = 1$; $f$ has no local extremum at $x = 2$; $f$ has a local maximum at $x = 3$.

c. $f'(x)$ is undefined when $x = 1$ and 3; $f''(2) = 0$; $f$ has a local maximum at $x = 1$; $f$ has a local minimum at $x = 2$; $f$ has an absolute maximum at $x = 3$; $f$ has an absolute minimum at $x = 4$.

Solution: In each case, we give just one possible solution; there are many correct possible answers. The answer for part a. is given in red, that for part b. is given in blue, and that for part c. is given in green.

5. (Section 4.1, exercises 37, 41, 45.) For each of the following functions $f$:

i. Find the critical point(s) of $f$ on the given interval;

ii. Determine the absolute extreme values of $f$ on the given interval when they exist;

iii. Use a graphing utility to confirm your conclusions.

a. $f(x) = x^2 - 10$ on $[-2, 3]$.

b. $f(x) = \sin(3x)$ on $[-\pi/4, \pi/3]$.

c. $f(x) = x^2 + \arccos x$ on $[-1, 1]$.

Solution: For each function, we will differentiate it, determine where the derivative vanishes or fails to exist, and then evaluate the function at each of these critical points (when possible) and at the endpoints of the intervals.
a. We have $f'(x) = 2x$, which is a polynomial and therefore defined everywhere; clearly, it only vanishes when $x = 0$, so this is the only critical point. Evaluating at $x = -2, 0,$ and $3$ gives

$$f(-2) = (-2)^2 - 10 = -6, \quad f(0) = 0^2 - 10 = -10, \quad f(3) = (3)^2 - 10 = -1,$$

and we see that the absolute maximum of $f$ occurs at $x = 3$, and the absolute minimum occurs at $x = 0$. The graph is given below.

b. By the chain rule, we have $f'(x) = 3 \cos(3x)$, which is defined everywhere. We know that $\cos x$ is zero for half-integer multiples of $\pi$ (i.e. when $x$ equals $\pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \ldots$), and so $\cos(3x)$ is zero when $x \in \{\pm \frac{\pi}{6}, \pm \frac{\pi}{2}, \pm \frac{5\pi}{6}, \ldots\}$.

Looking only at the values in the interval $[-\pi/4, \pi/3]$, we end up with the critical points $x = \pm \frac{\pi}{6}$. Evaluating $f$ at these points gives us

$$\sin\left(3 \cdot \frac{\pi}{6}\right) = 1, \quad \sin\left(3 \cdot \frac{-\pi}{6}\right) = -1,$$

and so these are the absolute extrema attained by the function on this interval. The graph is given below.

c. Remembering our derivatives of inverse trigonometric functions, we get

$$f'(x) = 2x - \frac{1}{\sqrt{1-x^2}} \text{ on } (-1, 1).$$
This function fails to be defined when $x = \pm 1$, and we see that

$$f'(x) = 0 \iff 2x = \frac{1}{\sqrt{1-x^2}} \iff 2x\sqrt{1-x^2} = 1.$$  

Squaring both sides, we see that $f'(x) = 0$ implies that $4x^2(1-x^2) = 1$, from which we get the equation

$$4x^4 - 4x^2 + 1 = 0,$$

or $4t^2 - 4t + 1 = 0$ when $t = x^2$.

The equation $4t^2 - 4t + 1$ factors as $(2t - 1)^2$, and so

$$4t^2 - 4t + 1 = 0 \iff (2t - 1)^2 = 0 \iff 2t = 1,$$

and so $t = x^2 = \frac{1}{2}$ is our solution. Because $x = -\frac{1}{\sqrt{2}}$ does not satisfy the equation $f'(x) = 0$ (check this!), we have that our only critical points are $\pm 1$ and $\frac{1}{\sqrt{2}}$. Evaluating at these points, we have

$$f(-1) = (-1)^2 + \arccos(-1) = 1 + \pi, \quad f(1) = 1^2 + \arccos(1) = 1 + 0 = 1,$$

$$f \left( \frac{1}{\sqrt{2}} \right) = \frac{1}{2} + \arccos \left( \frac{1}{\sqrt{2}} \right) = \frac{1}{2} + \frac{\pi}{4}.$$  

We have the inequalities

$$f(-1) < f \left( \frac{1}{\sqrt{2}} \right) < f(1)$$

and we see that the absolute maximum occurs at $x = 1$, and the absolute minimum occurs at $x = -1$. We give the graph below.

6. (Section 4.1, exercise 77.) You must get from a point $P$ on the straight shore of a lake to a stranded swimmer who is $50m$ from a point $Q$ on the shore that is $50m$ from you (see figure 2.) If you can swim at a speed of $2m/s$ and run at a speed of $4m/s$, at what point along the shore (say, $x$ metres from $Q$) should you stop running and start swimming if you want to reach the swimmer in the minimum time?

a. Find the function $T$ that gives the travel time as a function of $x$, where $0 \leq x \leq 50$.

b. Find the critical point(s) of $T$ on the open interval $(0, 50)$. 
c. Evaluate $T$ at the critical point and the endpoints ($x = 0$ and $x = 50$) to verify that the critical point corresponds to an absolute minimum. What is the minimum travel time?

d. Graph the function $T$ to check your work.

**Solution:** We will refer to figure 2 in our solution.

a. Per the figure, the distance to be walked (in red) is $50 - x$ metres, and by the Pythagorean theorem, the distance to be swum is \( \sqrt{50^2 + x^2} = \sqrt{2500 + x^2} \) metres. Dividing these distances by the respective speed at which you can run/swim, we obtain the formula

\[
T(x) = \frac{50 - x}{4 \text{ m/s}} + \frac{\sqrt{2500 + x^2}}{2 \text{ m/s}} = \frac{25}{2} - \frac{x}{4} + \frac{\sqrt{2500 + x^2}}{2} \text{ seconds.}
\]

b. We begin by differentiating: we have

\[
T'(x) = -\frac{1}{4} + \frac{x}{2\sqrt{2500 + x^2}},
\]

which is defined for all $x$ in our domain (i.e. $x \in [0, 50]$). It equals zero precisely when

\[
\frac{x}{2\sqrt{2500 + x^2}} = \frac{1}{4} \iff 2x = \sqrt{2500 + x^2} \iff 4x^2 = 2500 + x^2,
\]

and the last equation holds if and only if $3x^2 = 2500$, if and only if $x^2 = \frac{2500}{3}$. Because we know $x$ must be positive, we deduce that $x = \sqrt{\frac{2500}{3}} = \frac{50}{\sqrt{3}}$ is our sole critical point.

c. Per our work in parts a. and b., we have

\[
T(0) = \frac{25}{2} - 0 + \frac{\sqrt{2500 + 0^2}}{2} = \frac{25}{2} + \frac{50}{2} = \frac{75}{2} = 37.5,
\]

\[
T(50) = \frac{50 - 50}{4} + \frac{\sqrt{2500 + 2500}}{2} = \frac{50\sqrt{2}}{2} = 25\sqrt{2} = 35.355339\ldots,
\]

and finally

\[
T\left(\frac{50}{\sqrt{3}}\right) = \frac{25}{2} - \frac{50}{4\sqrt{3}} + \frac{\sqrt{2500 + \left(\frac{2500}{3}\right)}}{2} = \frac{25}{2} - \frac{25}{2\sqrt{3}} + \frac{50\sqrt{4/3}}{2}
\]

\[
= \frac{25}{2} - \frac{25}{2\sqrt{3}} + \frac{50}{\sqrt{3}} = \frac{75}{2\sqrt{3}} + \frac{25}{2} = 34.1506\ldots
\]

Evidently the minimum travel time is about 34.1506 seconds.
d. We give the graph of $T(x)$ below; the minimum time is marked.

7. \textit{(Section 4.2, exercises 39, 41, 47.)} For each of the following functions $f$:

i. Locate the critical point(s) of $f$;

ii. Use the First Derivative Test to locate the local maximum and minimum values;

iii. Identify the absolute maximum and minimum values of the function on the given interval (when they exist).

a. $f(x) = x^2 + 3$ on $[-3, 2]$.

b. $f(x) = x\sqrt{4 - x^2}$ on $[-2, 2]$.

c. $f(x) = \sqrt{x}\ln x$ on $(0, \infty)$.

\textbf{Solution:}

a. Clearly $f'(x) = 2x$ is a polynomial and so is defined everywhere, and $f'(x) = 0$ if and only if $x = 0$; so this is the only critical point of $f$ on this domain. Furthermore, $f'(x)$ changes sign from negative to positive at $x = 0$, and so by the First Derivative Test this point is a local minimum of $f$.

To check absolute extrema, it remains only to calculate the value of $f$ at the endpoints; we have

$$f(-3) = (-3)^2 + 3 = 12, \quad f(2) = 2^2 + 3 = 7, \quad f(0) = 0^2 + 3 = 3,$$

and we conclude that the absolute maximum occurs at $x = -3$ and that the absolute minimum occurs at $x = 0$.

b. By the product rule, we have that

$$f'(x) = \sqrt{4 - x^2} + x\left(\frac{-x}{\sqrt{4 - x^2}}\right) = \sqrt{4 - x^2} - \frac{x^2}{\sqrt{4 - x^2}}.$$
This function fails to be defined at $x = \pm 2$, so these are critical points. Moreover,
\[
f'(x) = 0 \iff \sqrt{4-x^2} = \frac{x^2}{\sqrt{4-x^2}} \iff 4-x^2 = x^2 \iff 2x^2 = 4,
\]
and so $x = \pm \sqrt{2}$; so these are the other critical points.
To apply the First Derivative Test, we compute the three values
\[
f'(\pm \sqrt{3}) = \sqrt{4-3} - \frac{3}{\sqrt{4-3}} = -2, \quad f'(0) = \sqrt{4-0} - \frac{3}{\sqrt{4-0}} = \frac{1}{2},
\]
and we see that $f'$ goes from negative to positive at $x = -\sqrt{2}$, and from positive to negative at $x = \sqrt{2}$; thus the former is the location of a local minimum, and the latter is the location of a local maximum.
We compute the values of $f$ at the critical points:
\[
f(-2) = (-2)\sqrt{4-2^2} = -f(2) = 0, \quad f(-\sqrt{2}) = (-\sqrt{2})\sqrt{4-2} = 2 = -f(\sqrt{2}),
\]
and we see that the absolute minimum value is $-2$ and the absolute maximum value is 2.

c. By the product rule,
\[
f'(x) = \sqrt{x} \left( \frac{1}{x} \right) + \ln x \left( \frac{1}{2\sqrt{x}} \right) = \frac{1}{\sqrt{x}} \left( 1 + \frac{1}{2} \ln x \right);
\]
this function is defined wherever $f(x)$ is defined. Furthermore, if $x > 0$, then
\[
f'(x) = 0 \iff 1 + \frac{1}{2} \ln x = 0 \iff \ln x = -2 \iff x = e^{-2},
\]
so this is the unique critical point of $f$ on our domain.
The sign of the derivative is always the same as the sign of $1 + \frac{1}{2} \ln x$ (because $\sqrt{x}$ is always positive here), and evidently $1 + \frac{1}{2} \ln x$ changes sign from negative to positive at our critical point. By the First Derivative test, there is a local minimum at $x = e^{-2}$; because our interval does not include its endpoints, this must therefore be the location of its absolute minimum.
Finally, we compute the value of $f$ at this point:
\[
f(e^{-2}) = \sqrt{e^{-2}} \ln(e^{-2}) = (e^{-1})(-2) = \frac{-2}{e},
\]
and so this is the absolute minimum value attained by $f$ for $x > 0$; it has no maxima, local or absolute.

8. (Section 4.2, exercises 57, 65, 73, 79.)
a. Determine the intervals on which the following functions are concave up or concave down. Identify any inflection points.
   i. $f(x) = x^4 - 2x^3 + 1$. 

b. Locate the critical points of the following functions. Then use the Second Derivative Test to determine (if possible) whether they correspond to local maxima or local minima.

i. \( f(x) = 4 - x^2 \).

ii. \( g(x) = x^2 e^{-x} \).

Solution:

a. We have

\[ f'(x) = 4x^3 - 6x, \quad f''(x) = 12x^2 - 6 = 6(2x^2 - 1) = -6(1 + \sqrt{2}x)(1 - \sqrt{2}x). \]

From its factored form we can see that \( f'' \) changes sign from positive to negative at \( x = -\frac{1}{\sqrt{2}} \), and then back to positive at \( x = \frac{1}{\sqrt{2}} \). It follows that \( f \) has inflection points at precisely \( x = \pm \frac{1}{\sqrt{2}} \), and that it is concave down on the interval \((-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})\), and is concave up outside this interval.

We proceed identically for \( g(x) \): the chain rule gives us

\[ g'(x) = e^{-x^2/2}(-2x/2) = -xe^{-x^2/2} = -xg(x), \]

and the product rule gives us

\[ g''(x) = -g(x) + (-x)g'(x) = -g(x) - x(-xg(x)) = g(x)(x^2 - 1), \]

that is,

\[ g''(x) = e^{-x^2/2}(x^2 - 1). \]

Because the exponential term is always positive, it is clear that the sign of \( g'' \) (and therefore the concavity of \( f \)) is determined by the sign of \( x^2 - 1 \), which is negative on \((-1, 1)\) and is positive for \(|x| > 1\).

It follows from these remarks that \( g \) has inflection points at \( x = \pm 1 \), and that it is concave down on the interval \((-1, 1)\) and is concave up outside this interval.

b. We begin by differentiating. We have \( f'(x) = -2x \), which exists everywhere and vanishes only at \( x = 0 \); so this is the only critical point. The second derivative is the constant function \( f''(x) = -2 \), and so in particular \( f''(0) < 0 \). It follows from the Second Derivative Test that \( x = 0 \) corresponds to a local maximum of \( f(x) \).

We do the same thing for \( g(x) \); the product rule gives us

\[ g'(x) = 2xe^{-x} + x^2e^{-x}(-1) = 2xe^{-x} - x^2e^{-x} = 2xe^{-x} - g(x); \]

this function is defined everywhere and vanishes exactly when

\[ g(x) = 2xe^{-x} \iff x^2e^{-x} = 2xe^{-x} \iff x = 0 \text{ or } x = 2, \]

and so these are the critical points of \( g(x) \). We compute the second derivative:

\[
\begin{align*}
g''(x) &= 2e^{-x} + 2xe^{-x}(-1) - g'(x) \\
&= 2e^{-x} - 2xe^{-x} - (2xe^{-x} - x^2e^{-x}) \\
&= e^{-x}(x^2 - 4x + 2).
\end{align*}
\]
Again, the sign of this function is the same as that of \(x^2 - 4x + 2\), which the Pythagorean theorem tells us has roots at

\[
x = \frac{-(4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(1)} = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}.
\]

In particular, this means that

\[
f''(x) < 0 \iff 2 - \sqrt{2} < x < 2 + \sqrt{2},
\]

and so \(f''(0) > 0\) while \(f''(2) < 0\). We deduce by the Second Derivative Test that \(x = 0\) is the location of a local minimum while \(x = 2\) is the location of a local maximum.

9. (Section 4.2, exercise 95.) The graph of \(f'\) on the interval \([-3, 2]\) is shown in figure 3.

a. On what interval(s) is \(f\) increasing? decreasing?

b. Find the critical points of \(f\). Which critical points correspond to local maxima? local minima? neither?

c. At what point(s) does \(f\) have an inflection point?

d. On what interval(s) is \(f\) concave up? concave down?

e. Sketch the graph of \(f''\).

f. Sketch one possible graph of \(f\).

**Solution:**

a. We know that \(f\) is increasing precisely when \(f' > 0\), and conversely that \(f\) is decreasing when \(f' < 0\). According to the figure, we can say that \(f'(x) < 0\) when \(-3 \leq x < -2\), and that \(f'(x) > 0\) for all \(-1 \leq x \leq 2\) except for \(x = 0\), when \(f' = 0\). It follows that \(f\) decreases from \(-3\) to \(-2\), and increases on both intervals \((-2, 0)\) and \((0, 2)\).

b. By definition, a critical point of \(f\) is a point at which \(f'\) is not defined, or equals zero. Because our graph of \(f'\) is defined everywhere, we need only look at where \(f' = 0\); from the graph, this appears to be at \(x = -2\) and \(x = 0\).

When \(x = -2\), the function \(f'\) changes sign from negative to positive, meaning that \(f\) has stopped decreasing, and has started to increase; we deduce that \(f\) has a local minimum at \(x = -2\). At \(x = 0\), \(f'\) does not change sign, going from positive, to zero, to positive. It follows that \(f\) has neither local maximum nor local minimum here.

c. An inflection point can only occur when \(f'' = 0\) or when \(f''\) fails to exist. From the graph, we see that the line tangent to \(y = f'\) fails to exist when \(x = -1\), and is horizontal when \(x = 0\); we deduce that these are possible inflection points. To confirm this, we must check that \(f''\) changes sign at each point.

Between \(-3\) and \(-1\), the graph of \(f'\) is linear and so equals its own tangent line; because this line is increasing, we deduce that \(f''\) is positive on this interval. Between \(-1\) and 0 the slope of the tangent line to the graph of \(f'\) has negative slope, so \(f''\) must be
negative here (note that there will be a jump discontinuity in $f''$ at $x = -1$.) Thus $x = -1$ is indeed an inflection point.

Finally we observe that between 0 and 1, the slope of the tangent line to the graph of $f'$ has positive slope and so $f''$ is positive here. By continuity, $f''$ will pass through zero as it goes from negative to positive (as $x$ increases from a negative quantity to a positive one), and so $x = 0$ is also an inflection point.

d. We know that intervals where $f'' > 0$ correspond to regions where the graph of $f$ is concave up, and conversely that regions where $f'' < 0$ correspond to regions where the graph of $f$ is concave down.

Our remarks in part c. imply that $f'' > 0$ on the open intervals $(-3, -1)$ and $(0, 4)$, and so these are the intervals on which $f$ is concave up; similarly we have shown that $f'' < 0$ on the open interval $(-1, 0)$, and so this is the interval on which $f$ is concave down.

e. The graph of $f''$ is given below.

f. One possible graph of $f$ is given below; all correct answers will have this shape, but can be shifted up or down by adding constants.