All questions come from the course text. Problems 1–3 cover related rates, problems 4–6 cover optimization and extrema, and problems 7–9 cover second derivatives and applications.

1. (Section 3.11, exercise 5.) The sides of a square increase in length at a rate of 2 m/s.
   a. At what rate is the area of the square changing when the sides are 10 m long?
   b. At what rate is the area of the square changing when the sides are 20 m long?
   c. Draw a graph that shows how the rate of change of the area varies with the side length.

2. (Section 3.11, exercise 19.) A swimming pool is 50 m long and 20 m wide. Its depth decreases linearly along the length from 3 m to 1 m (from the deep end to the shallow end). It is initially empty and is filled in at a rate of 1 m³/min. How fast is the water level rising 4 hours after the filling begins? How long will it take to fill the pool?

3. (Section 3.11, exercise 29.) An inverted conical water tank with a height of 12 feet and a radius of 6 feet is drained through a hole in the vertex at a rate of 2 cubic feet per second (see figure 1). What is the rate of change of the water depth when the water depth is 3 feet? (Hint: Use similar triangles.)

4. (Section 4.1, exercises 19–21.) Sketch a graph of a function $f$ that is continuous on $[0, 4]$ and satisfies the given properties.
   a. $f'(x) = 0$ when $x = 1$ and 2; $f$ has an absolute maximum at $x = 4$; $f$ has an absolute minimum at $x = 0$; $f$ has a local minimum at $x = 2$.
   b. $f'(x) = 0$ when $x = 1, 2,$ and 3; $f$ has an absolute minimum at $x = 1$; $f$ has no local extremum at $x = 0$; $f$ has a local maximum at $x = 3$.
   c. $f'(x)$ is undefined when $x = 1$ and 3; $f'(2) = 0$; $f$ has a local maximum at $x = 1$; $f$ has an local minimum at $x = 2$; $f$ has an absolute maximum at $x = 3$; $f$ has an absolute minimum at $x = 4$.

5. (Section 4.1, exercises 37, 41, 45.) For each of the following functions $f$:
   i. Find the critical point(s) of $f$ on the given interval;
   ii. Determine the absolute extreme values of $f$ on the given interval when they exist;
   iii. Use a graphing utility to confirm your conclusions.
   a. $f(x) = x^2 - 10$ on $[-2, 3]$.
   b. $f(x) = \sin(3x)$ on $[-\pi/4, \pi/3]$.
   c. $f(x) = x^2 + \arccos x$ on $[-1, 1]$.

6. (Section 4.1, exercise 77.) You must get from a point $P$ on the straight shore of a lake to a stranded swimmer who is 50 m from a point $Q$ on the shore that is 50 m from you (see figure.) If you can swim at a speed of 2 m/s and run at a speed of 4 m/s, at what point along the shore (say, $x$ metres from $Q$) should you stop running and start swimming if you want to reach the swimmer in the minimum time?
   a. Find the function $T$ that gives the travel time as a function of $x$, where $0 \leq x \leq 50$.
   b. Find the critical point(s) of $T$ on the open interval $(0, 50)$.
   c. Evaluate $T$ at the critical point and the endpoints ($x = 0$ and $x = 50$) to verify that the critical point corresponds to an absolute minimum. What is the minimum travel time?
d. Graph the function $T$ to check your work.

7. (Section 4.2, exercises 39, 41, 47.) For each of the following functions $f$:
   i. Locate the critical points of $f$;
   ii. Use the First Derivative Test to locate the local maximum and minimum values;
   iii. Identify the absolute maximum and minimum values of the function on the given interval (when they exist).
   a. $f(x) = x^2 + 3$ on $[-3, 2]$.
   b. $f(x) = x\sqrt{4 - x^2}$ on $[-2, 2]$.
   c. $f(x) = \sqrt{x}\ln x$ on $(0, \infty)$.

8. (Section 4.2, exercises 57, 65, 73, 79.)
   a. Determine the intervals on which the following functions are concave up or concave down. Identify any inflection points.
      i. $f(x) = x^4 - 2x^3 + 1$.
      ii. $f(x) = e^{-x^2/2}$.
   b. Locate the critical points of the following functions. Then use the Second Derivative Test to determine (if possible) whether they correspond to local maxima or local minima.
      i. $f(x) = 4 - x^2$.
      ii. $f(x) = x^2e^{-x}$.

9. (Section 4.2, exercise 95.) The graph of $f'$ on the interval $[-3, 2]$ is shown in figure 3.
   a. On what interval(s) is $f$ increasing? decreasing?
   b. Find the critical points of $f$. Which critical points correspond to local maxima? local minima? neither?
   c. At what point(s) does $f$ have an inflection point?
   d. On what interval(s) is $f$ concave up? concave down?
   e. Sketch the graph of $f''$.
   f. Sketch one possible graph of $f$.