All questions come from the course text. Recall the formula for compound interest: a principal amount of $P$ invested at an annual interest rate $r$, compounded $n$ times per year, will accrue to the amount

$$A(t) = P \left(1 + \frac{r}{n}\right)^{nt}$$

after $t$ years. A principal amount of $P$ invested at an annual interest rate $r$ which is compounded continuously over $t$ years will accrue to the amount

$$A(t) = Pe^{rt},$$

where $e = 2.718281828\ldots$ is the base of the natural logarithm.

1. One hundred grams of a particular radioactive substance decays according to the function $M(t) = 100e^{-t/650}$, where $t > 0$ measures time in years. When does the mass reach 50 grams? (Section 1.3, exercise 57.)

2. A culture of bacteria has a population of 150 cells when it is first observed. The population doubles every 12 hours, which means that its population is governed by the function $p(t) = 150 \cdot 2^{t/12}$, where $t$ is the number of hours after the first observation.
   a. Verify that $p(0) = 150$, as claimed.
   b. Show that the population doubles every 12 hours, as claimed.
   c. What is the population 4 days after the first observation?
   d. How long does it take for the population to triple in size?
   e. How long does it take for the population to reach 10,000 cells? (Section 1.3, exercise 79.)

3. (Graphing utility required.) A capacitor is a device that stores electrical charge. The charge on a capacitor accumulates according to the function $Q(t) = a(1 - e^{-t/c})$, where $t$ is measured in seconds and $a > 0$ and $c > 0$ are physical constants. The steady-state charge is the value that $Q(t)$ approaches as $t$ becomes large.
   a. Graph the charge function for $t \geq 0$ using $a = 1$ and $c = 10$. Find a graphing window that shows the full range of the function.
   b. Vary the value of $a$ while holding $c$ fixed. Describe the effect on the curve. How does the steady-state charge vary with $a$?
   c. Vary the value of $c$ while holding $a$ fixed. Describe the effect on the curve. How does the steady-state charge vary with $c$?
   d. Find a formula that gives the steady-state charge in terms of $a$ and $c$. (Section 1.3, exercise 80.)

4. The half-life of an antibiotic in the bloodstream is 10 hours. If an initial dose of 20 milligrams is administered, the quantity left after $t$ hours is modeled by $Q(t) = 20e^{-0.0693t}$, for $t \geq 0$.
   a. Find the instantaneous rate of change of the amount of antibiotic in the bloodstream, for $t \geq 0$.
   b. How fast is the amount of antibiotic changing at $t = 0$? at $t = 2$?
c. Evaluate and interpret \( \lim_{t \to \infty} Q(t) \) and \( \lim_{t \to \infty} Q'(t) \).

\((\text{Section 3.4, exercise 53.})\)

5. A $200 investment in a savings account grows according to \( A(t) = 200e^{0.0398t} \), for \( t \geq 0 \), where \( t \) is measured in years.

a. Find the balance of the account after 10 years.

b. How fast is the account growing (in dollars per year) at \( t = 10 \)?

c. Use your answers to parts a. and b. to write the equation of the line tangent to the curve \( A(t) = 200e^{0.0398t} \) at the point \( (10, A(10)) \).

\((\text{Section 3.4, exercise 54.})\)

6. Iodine-123 is a radioactive isotope used in medicine to test the function of the thyroid gland. If a 350 microcurie (\( \mu \text{Ci} \)) dose of iodine-123 is administered to a patient, the quantity \( Q = Q(t) \) left in the body after \( t \) hours is approximately \( Q(t) = 350 \left( \frac{1}{2} \right)^{t/13.1} \).

a. How long does it take for the level of iodine-123 to drop to 10\( \mu \text{Ci} \)?

b. Find the rate of change of the quantity of iodine-123 at 12 hours, at 1 day, and at 2 days. What do your answers say about the rate at which iodine decreases as time increases?

\((\text{Section 3.9, exercise 33.})\)

7. Beginning at age 30, a self-employed plumber saves $250 per month in a retirement account until she reaches age 65. The account offers 6% interest, compounded monthly. The balance in the account after \( t \) years is given by \( A(t) = 50,000(1.005^{12t} - 1) \).

a. Compute the balance in the account after 5, 15, 25, and 35 years. What is the average rate of change in the value of the account over the intervals [5, 15], [15, 25], and [25, 35]? 

b. Suppose the plumber started saving at age 25 instead of at age 30. Find the balance at age 65 (i.e. after 40 years of investing).

c. Use the derivative \( \frac{dA}{dt} \) to explain the surprising result in part b. and to explain this advice: Start saving for retirement as early as possible.

\((\text{Section 3.9, exercise 97.})\)

8. The U.S. government reports the rate of inflation (as measured by the Consumer Price Index) both monthly and annually. Suppose that for a particular month, the monthly rate of inflation is reported as 0.8%. Assuming that this rate remains constant, what is the corresponding annual rate of inflation? Is the annual rate 12 times the monthly rate? Explain. \((\text{Section 6.9, exercise 37.})\)