1. Explain the intermediate value theorem using words and pictures. (Exercise 8.)

2. Determine the points at which the following function \( f \) has discontinuities. At each point of discontinuity, state the conditions in the continuity checklist that are violated. (Exercise 10.)

3. Let
\[
f(x) = \begin{cases} 
\frac{x^2 - 4x + 3}{x - 3} & \text{if } x \neq 3, \\
2 & \text{if } x = 3. 
\end{cases}
\]
Determine whether or not \( f(x) \) is continuous at \( a = 3 \). Use the continuity checklist to justify your answer. (Exercise 18.)

4. Determine the interval(s) on which the function \( f(x) = \frac{x^5 + 6x + 17}{x^2 - 9} \) is continuous (Hint: use theorem 2.10 from the textbook.) (Exercise 23.)

5. Let
\[
f(x) = \begin{cases} 
2x & \text{if } x < 1, \\
x^2 + 3x & \text{if } x \geq 1. 
\end{cases}
\]
   a. Use the continuity checklist to show that \( f \) is not continuous at 1.
   b. Is \( f \) continuous from the left or from the right at 1?
   c. State the interval(s) of continuity.
   (Exercise 39.)

6. You are shopping for a $150,000, 30-year loan to buy a house. The monthly payment is
\[
m(r) = \frac{150000(r/12)}{1 - (1 + r/12)^{-360}},
\]
where \( r \) is the annual interest rate. Suppose banks are currently offering interest rates between 6% and 8%.
a. Use the intermediate value theorem to show that there is a value of \( r \) in the interval (0.06, 0.08) – i.e., an interest rate between 6% and 8% – that allows you to make monthly payments of $1000 per month.

b. Use a graph to illustrate your explanation to part a. Then determine the interest rate you need for monthly payments of $1000.

(Exercise 58.)

7. Determine whether or not the following statements are true, and provide an explanation or counterexample.

a. If a function is left-continuous and right-continuous at \( a \), then it is continuous at \( a \).

b. If a function is continuous at \( a \), then it is left-continuous and right-continuous at \( a \).

c. If \( a < b \) and \( f(a) \leq L \leq f(b) \), then there is some value of \( c \) in \((a, b)\) for which \( f(c) = L \).

d. Suppose \( f \) is continuous on \([a, b]\). Then there is a point \( c \) in \((a, b)\) such that \( f(c) = \frac{f(a) + f(b)}{2} \).

(Exercise 65.)

8. Determine the value of the constant \( a \) for which the function

\[
 f(x) = \begin{cases} 
 x^2 + 3x + 2 & \text{if } x \neq -1, \\
 a & \text{if } x = -1.
\end{cases}
\]

is continuous at \(-1\). (Exercise 84.)

9. Let

\[
 f(x) = \begin{cases} 
 x^2 + x & \text{if } x < 1, \\
 a & \text{if } x = 1, \\
 3x + 5 & \text{if } x > 1.
\end{cases}
\]

a. Determine the value of \( a \) for which \( g \) is continuous from the left at 1.

b. Determine the value of \( a \) for which \( g \) is continuous from the right at 1.

c. Is there a value of \( a \) for which \( g \) is continuous at 1? Explain why or why not.

(Exercise 85.)