Math 532: Algebraic Geometry I
Course outline

Instructor

Kai Behrend
Math Annex 1213
behrend@math.ubc.ca

Algebraic Geometry

In a nutshell, algebraic geometry is the study of geometry by algebraic means.

For example, conic sections are described by their equations, such as $x^2+y^2 = 1$. (This is a quadratic equation). Of course, we also consider higher order equations, such as cubics, $y^2 = x^3 + x + 1$. One of the most basic theorems in algebraic geometry is Bezout's theorem (18th century): the number of intersection points of two such algebraic curves in the plane, one of degree m and one of degree n is always nm. (for example, our above two curves should have 6 intersection points). There are many issues to be resolved, though, to make this always true. For example, some of the intersection points might have imaginary coordinates (this problem is resolved by working over the complex numbers). Or they might lie "at infinity". This is why we study "projective" geometry. (By the way, from the point of view of complex projective geometry all conic sections, ellipses, parabolas, hyperbolas look the same.) Finally, the curves might be tangent to each other, which means that such intersection points have 'multiplicity'. Bezout's theorem will be one of the highlights of the course.

As you can see, algebraic geometry is a very old subject, with a long history. But it is also a subject in which research is very active these days. Just recently (within the last 15 years), old questions such as 'how many (parameterizable) curves of degree d in the plane pass through 3d-1 given points?' have found answers, by the discovery of completely unexpected connections with quantum physics.

Algebraic geometry is also very important in number theory. The study of Diophantine equations (such as the solution of Fermat's last theorem) is impossible without algebraic geometry.

The Course

This course is intended to be a first introduction to algebraic geometry. The goal is to prepare students to take more advanced courses in the subject (such as the follow up course Math 533 in the Spring). We will study the basic objects of algebraic geometry, *algebraic varieties* and *schemes.* The course will loosely follow the lecture notes by Gathmann to be found at `http://www.mathematik.uni-kl.de/~gathmann/alggeom.php`.

We hope to cover Chapters 0 through 6.

Prerequisites

A one year graduate course in algebra is certainly highly desirable. A graduate course (or advanced undergraduate course) in differential geometry or topology would be advantageous, though not strictly necessary.

Homework

There will be sporadic homework assignments which you will be expected to complete. Your mark will be partially based on these.

Project

You will be expected to complete a project. Details will follow later. Your mark will be partially based on the project.

Grading

Grades will be based half on homework and half on the project. They will be in accordance with the math departments *grading policies for graduate courses*.

Textbooks

Besides the above mentioned notes by Gathmann, the following introductory books may be useful:

- *Algebraic Curves: An introduction to Algebraic Geometry* by William Fulton. Addison Wesley, 1974.

- *The red book of Varieties and Schemes* by David Mumford. Springer Lecture Notes, 1999.

- *Algebraic Varieties* by George R. Kempf. Cambridge University Press, 1993.

Other books which I recommend are listed below. They are all quite different from each other in style, but they are all very useful for more in-depth study of the subject.

- *Algebraic Geometry* by Joe Harris. Springer Verlag. Graduate Texts in Mathematics 133.
  This is a good supplement to the main text with many more examples and quite a different style.

- *The Geometry of Schemes* by David Eisenbud and Joe Harris. Springer Verlag. Graduate Texts in Mathematics 197.
  This goes beyond algebraic varieties by introducing schemes, the most commonly used tool in modern algebraic geometry.

- *Algebraic Geometry* by Robin Hartshorne. Springer Verlag. Graduate Texts in Mathematics 52.
  This is the classic textbook of algebraic geometry today. It is very carefully written and has a huge number of challenging exercises. The main subject of this book is, again, the theory of schemes, but in much more detail than is suitable for a first course.

- *Commutative Algebra* by David Eisenbud. Springer Verlag. Graduate Texts in Mathematics 150.
  This is a somewhat chatty but very exhaustive treatment of all the algebra you are likely to encounter in this course.

- *Commutative Ring Theory* by H. Matsumura. Cambridge University Press. Cambridge Studies In Advanced Mathematics 8.
  This is an extremely concise treatment of much of the algebra you are going to encounter in algebraic geometry.

- *An Algebraic Introduction To Complex Projective Geometry. I. Commutative Algebra* by Christian Peskine. Cambridge University Press. Cambridge Studies In Advanced Mathematics 47.
  This is an introduction to commutative algebra and highly recommended if your algebra background is weak.