Midterm Exam

Friday, October 18, 2013

No books, notes or calculators

Problem 1.
Define the following terms:
(a) stem field of an irreducible polynomial,
(b) splitting field of a polynomial,
(c) algebraic field extension,
(d) finite separable field extension,

Problem 2.
Carefully state the following results:
(a) the Eisenstein irreducibility criterion
(b) an algebraic criterion for when a real number is constructible

Problem 3.
Give examples of the following phenomena (without proofs):
(a) An irreducible polynomial in $\mathbb{Q}[x]$, of degree at least 2, whose stem field coincides with its splitting field,
(b) a field extension of degree 100,
(c) an inseparable field extension,
(d) an irreducible polynomial in $\mathbb{F}_2[x]$ of degree at least 3,

Give proofs:

Problem 4.
Find the degree of the splitting field of $x^4 + 4$ over $\mathbb{Q}$.

Problem 5.
Find the degrees of the splitting fields of $x^7 - 1$ and $x^8 - 1$ over $\mathbb{F}_4$, the field with 4 elements.

Problem 6.
Let $|K(\alpha) : K| = 5$. Prove that $K(\alpha^2) = K(\alpha)$.

Problem 7.
Find the minimal polynomial of $\sqrt{6} + \sqrt{-1}$ over $\mathbb{Q}$.