Groupoids and Stacks in Physics and Geometry
June 28 – July 2, 2004
CIRM Luminy

Abstracts

Anton Alekseev
Deformation quantization of coadjoint orbits

I will explain how Fedosov's quantization construction applies to coadjoint orbits. In many cases, one obtains an explicit formula for invariant $*$-products. This talk is based on joint work with A. Lachowska and A. Szenes.

Clemens Berger
Higher groupoids and homotopy type

In his pursuit of stacks, A. Grothendieck formulated the conjecture that a weak form of n-groupoid should serve as algebraic model for the homotopy n-type of an arbitrary topological space. The need of a well-founded concept of n-category has recently emphasized the importance of this conjecture. In this talk, I try to explain how a suitable abstract homotopy theory for spaces may generate models for homotopy n-types which resemble very much weak n-groupoids. The underlying combinatorics are based on a higher analog of the simplex category, which has been introduced by A. Joyal in order to capture the combinatorial structure of the topological n-ball.

Jonathan Block
Noncommutative abelian varieties and nonabelian varieties

We will discuss some aspects of the role of stacks in noncommutative geometry.

Peter Bouwknegt
T-duality and topology change

T-duality, in its simplest form, is the R to $1/R$ symmetry of String Theory compactified on a circle of radius R. It can be generalized to manifolds which admit circle actions (e.g. circle bundles) or, more generally, torus actions. In the case of nontrivial torus bundles, and in the background of H-flux, T-duality relates manifolds of different topology and in particular provides isomorphisms between the twisted cohomologies and twisted K-theories of these manifolds. In this talk we will discuss these developments as well as provide some examples.

Lawrence Breen
Gerbes and 2-groupoids

We review the notions of a gerbe and of a 2-groupoid, and discuss the relation between them.

Alberto Cattaneo
Coisotropic submanifolds and dual pairs

Coisotropic submanifolds play a fundamental role in symplectic and Poisson geometry as they describe systems with symmetries ('first-class constraints') and provide a method to generate new symplectic or Poisson spaces ('symplectic or Poisson reduction').

Though often singular, these reduced spaces possess anyway a Poisson algebra of functions that is defined as a quotient from the Poisson algebra of the original Poisson manifold.

Coisotropic submanifolds are also the natural boundary conditions for the Poisson sigma model, a field theory whose quantization leads, for instance, to Kontsevich's formula for deformation quantization.

In this talk I will discuss how symplectic reduction related to the Poisson sigma model with coisotropic boundary condition leads to (singular) dual pairs for the reduced Poisson manifolds.

Time permitting, I will shortly address the problem of quantizing these structures (by deformation).

Ezra Getzler

L_∞-algebras and n-groupoids

I will present a generalization of the Lie group associated to Lie algebras, in the context of (nilpotent) differential graded Lie algebras. My construction generalizes a number of other known functors: the Deligne groupoid, the Dold-Kan functor, ... In this way, I obtain the first clear picture of how Quillen's theory of nilpotent rational homotopy types is a generalization of Lie's correspondence for nilpotent Lie algebras.

Yvette Kosmann-Schwarzbach

The infinitesimals of Lie groupoids: Lie algebroids

We shall briefly review the main facts in the general theory of Lie algebroids, and new results on their cohomology due to Crainic and Moerdijk. Then we shall give examples, including some which were recently discovered in the context of Poisson and Dirac geometry.

John Lott

Higher-degree analogs of the determinant line bundle

Given a smooth family of Dirac-type operators on an odd-dimensional closed manifold, we construct an abelian gerbe-with-connection whose curvature is the 3-form component of the Atiyah-Singer families index theorem. More generally, given a smooth family of Dirac-type operators whose index lies in the i-th filtration of the reduced K-theory of the parametrizing space, we construct a set of Deligne cohomology class of degree i whose curvatures are the i-form component of the Atiyah-Singer families index theorem.

Kirill MacKenzie

Gerbes avant la lettre

I will speak on two matters which are, I believe, closely related to different aspects of work on gerbes:

1. There is an equivalence between extensions of Lie groupoids and structures which are, in a compatible way, both Lie groupoids and principal bundles ('PBG–groupoids'). This equivalence cannot be formulated in terms of the corresponding principal bundles, and is not restricted to structures with group S^1. There is a corresponding infinitesimal theory.

2. The construction of the cohomological obstruction to the integrability of a transitive Lie algebroid has many features which suggest it may be regarded in gerbe theoretic terms.

Charles-Michel Marle

The cotangent groupoid to a semi-direct product of Lie groupoids

The symplectic actions of two Lie groups on the same symplectic manifold are said to be completely orthogonal if, for any point x in that manifold, the tangent space at x to the orbit of one ot these actions is the symplectic orthogonal of the tangent space at x to the orbit of the other action. As a well known example, in which the orbits of the two actions are of the same dimension (half the dimension of the symplectic manifold), we have the canonical lifts to the cotangent bundle of the actions of a Lie group on itself by left and right translations. When that Lie group is a semi-direct product, symplectic reduction may be used to obtain completely orthogonal actions of two different Lie groups on the same reduced symplectic manifold, the orbits of these two actions being of different dimensions. For example, the phase space for the motion of a rigid body with a fixed point in a gravitational field, which is an orbit of the coadjoint action of the group of Euclidean displacements, can be obtained in that way. In my talk, I will generalize these properties by replacing Lie groups by Lie groupoids and the semi-direct product of a Lie group and a vector space by the semi-direct product of a Lie groupoid and a vector bundle.

Eckhard Meinrenken

Equivariant cohomology and the Maurer-Cartan *equation*

This talk is based on joint work with Anton Alekseev. Let G be a compact Lie group acting smoothly on a manifold M. The standard Cartan complex for the equivariant cohomology consists of equivariant polynomial maps from the Lie algebra of G into differential forms on M. In their 1998 paper, Goresky-Kottwitz-MacPherson described a 'small Cartan model', with underlying complex the *invariant* polynomials on the Lie algebra tensored with *invariant* differential forms. One of our main results gives an explicit quasi-isomorphism from the small Cartan model into the usual Cartan model. The construction involves the solution of an interesting Maurer-Cartan equation, and leads to a refinement of Chevalley's transgression theorem.

Jouko Mickelsson

Twisted K-theory invariants

An invariant for twisted K-theory classes on a compact 3-manifold is introduced. The invariant is then applied to the twisted K-theory classes arising from the supersymmetric $SU(2)$ Wess-Zumino model. It is shown that the classes defined by different highest weights of level k representations of the loop group $L(SU(2))$ are inequivalent, which is compatible with the Freed-Hopkins-Teleman correspondence between Verlinde algebra and twisted equivariant K-theory on compact Lie groups.

Jean Renault

The Dixmier-Douady invariant and continuous-trace C^*-algebras

I will review the Dixmier-Douady invariant in its original setting and will present various equivariant generalizations.

Pierre Schapira

Quantization-deformation modules on complex symplectic manifolds

The ring of microdifferential operators on the cotangent bundle of a complex manifold has been constructed by Sato-Kashiwara-Kawai in the 70's. Homogeneous symplectic transformations may be locally 'quantized' in order to operate on such rings and Kashiwara used this fact in 1996 to construct the stack of microdifferential modules on a contact complex manifold.

When considering a symplectic manifold X, one can similarly construct locally a sheaf of ring W_X, the ring of quantization-deformation operators (also called, WKB-differential operators) but the lack of homogeneity is replaced by a central parameter τ and W_{pt} is now a subfield k of $\mathbb{C}[\tau, \tau^{-1}]]$. (We work with analytic operators, although formal ones are more popular.)

In this talk, we first construct the stack of W-modules on X (joint work with P. Polesello) and classify simple W-modules along a smooth Lagrangian submanifold (joint work with A. D'Agnolo).

Next, (joint work with J-P. Schneiders), assuming X is compact, we prove finiteness and duality results over the field k for coherent W-modules. These results are similar to the classical ones by Cartan-Serre for O-modules over $\mathbb{C}$. We prove that the index formula is given by integrating a certain Euler class and we make a conjecture for a Riemann-Roch theorem in this setting.

Eric Sharpe

Some progress in the physics of string compactifications on stacks

Each presentation of a Deligne-Mumford stack of the form of a global quotient $[X/G]$ defines a G-gauged sigma model on X. For 'string compactifications on stacks' to be well-defined assumes that each such gauged sigma model *RG* flows to the same CFT, a very nontrivial physics statement. In this talk I will give a status report on work-in-progress with Tony Pantev on checking whether 'string compactifications on stacks' are indeed well-defined, concentrating on gerbes.

In the first part of this talk I will describe a physics proposal for quantum cohomology of certain gerbes, stemming from a description in terms of gauged linear sigma models. Whether these physical calculations have anything to do with gerbes, will be left to the audience to decide.

In the second part of this talk I will discuss different possible calculations of the massless spectrum of a closed string on a gerbe. One possibility, as a cohomology of the associated inertia stack, suffers from overcounting moduli. Another proposal correctly counts moduli, but the resulting physical theory is nonunitary and presentation-dependent.

Constantin Teleman

Tensor product K-theory and the stack of G-bundles over a curve

This talk will explain how a generalised cohomology theory, known to homotopy theorists as BU-tensor and related to twistings of K-theory, encodes the answer to the index problem over the moduli stack of G-bundles over a Riemann surface. Witten's integration formulae on the moduli space of stable bundles, originally derived from topological Yang-Mills theory, arise as large level limits of K-theory formulae. The surprisingly simple proof is joint work with C. Woodward.

Bertrand Toen

Moduli stack of dg-categories

The purpose of this talk is to report on some recent work (joint with G. Vezzosi) on moduli spaces of dg-categories. Dg-categories are a sort of categories for which morphisms have a natural structure of complexes which appear naturally in several contexts of algebraic geometry (minimal model, mirror symmetry…) have been considered by various authors as generalizations (or rather weakened versions) of the notion of schemes, and in particular it is excpected that they form moduli spaces. I this talk I will explain why algebraic spaces and stacks are not enough in order to construct moduli of dg-categories, and will present a generalization of those, called D-stacks, for which the construction makes sense.

Jean-Louis Tu

Twisted K-theory of groupoids

We review basic properties of the K-theory of C^*-algebras, particularly those coming from locally compact groupoids. Then, we define twisted K-theory of groupoids and review its basic properties. We recover the usual twisted K-theory of topological spaces, twisted equivariant K-theory, and the twisted K-theory of orbifolds.

Angelo Vistoli

Twisted curves and logarithmic structures

Twisted curves are algebraic stacks that occur naturally in several problems in moduli theory. When a smooth curve with some given structure (for example, a level structure) degenerates, the limit is often a twisted curve.

In the first part of the talk I will present the basic definition and illustrate it with some examples. In the second I will present a very concrete description of twisted curves via logarithmic structures, due to Martin Olsson.

Alan Weinstein

Integration of complex Lie algebroids

Lie algebroids in the smooth category are the infinitesimal objects corresponding to Lie groupoids. The notion extends easily to the holomorphic category if one defines the Lie algebroid operations on the sheaf of sections rather than on global sections.

In this talk, I will introduce the intermediate notion of 'complex Lie algebroid,' which is a complex vector bundle over a real manifold. There is not yet a general theory of these objects and the talk will mostly be devoted to problems and examples (including complex structures, generalized complex structures, and CR-structures) and questions. In particular, I will discuss the integrability problem, which is already nontrivial in the local sense, since it includes the problem of integrability of almost complex and CR-structures.

Chenchang Zhu

Integrating Lie algebroids via stacks

The infinitesimal data of a Lie groupoid is a Lie algebroid. But not every Lie algebroid can integrate to a Lie groupoid. In this talk, I will introduce Weinstein groupoids—the groupoids in the world of differentiable stacks. We will see that this little step from Lie groupoids (manifolds) to Weinstein groupoids (stacks) will complete the integration picture for Lie algebroids: the infinitesimal data of a Weinstein groupoid is a Lie algebroid and every Lie algebroid can integrate to a Weinstein groupoid whose infinitesimal data is this algebroid.