Quantum Cohomology of Stacks

February 12–16, 2007

Institut Henri Poincaré

Abstracts

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00		**L. Chen**		**Karp**	**Harada**
10:00	Coffee	Coffee		Coffee	Coffee
10:30	**Vistoli**	**Perroni**		**Corti**	**Chiodo**
11:45	**Coates**	**Tseng**		**W. Chen**	
15:00	**Goldin**	**Iritani**	**Mann**	**Kaufmann**	
16:00	Coffee	Coffee	Coffee	Coffee	
16:30	**Holm**	**Kimura**	**Ruan**	**Rose**	
19:30		Dinner			

Linda Chen
Enumerative geometry and Gromov-Witten theory of stacks

We give an approach to an enumerative geometry problem using Gromov-Witten theory of stacks. In particular, we compute the number of rational plane curves with prescribed tangencies to a smooth cubic. We show that certain invariants are enumerative, and obtain a formula for these numbers. This is joint work with Chuck Cadman.

Weimin Chen
Pseudoholomorphic curves in the context of orbifolds

One area of applications of Gromov's pseudoholomorphic curve theory is in the study of symplectic topology itself, which, for a number of reasons, was particularly powerful in the case of dimension 4. In this talk we will describe an extension of the theory to a context where spaces are allowed to have quotient singularities. Such singular spaces naturally occured, in particular, in problems involving actions of a compact Lie group. One basic conviction is that pseudoholomorphic curves in such a singular space encode certain global information about the singularities of the space, which is useful in various applications.

Alessandro Chiodo
Mumford formula generalised to rth roots: crepant and r-spin applications

I will present the generalisation [math/0607324] of Mumford's formula describing the Chern character of the Hodge bundle. The generalised formula describes the Chern character of the direct image of universal rth roots. It has two natural applications: to moduli of r-spin curves and to quantum cohomology of orbifolds.

Tom Coates
Mirror Symmetry and The Crepant Resolution Conjecture

I will explain how to use mirror symmetry to determine how genus-zero Gromov–Witten invariants of toric orbifolds are related to those of their crepant resolutions. A key ingredient is the genus-zero version of Givental's quantization formalism. This talk

is about joint work with Alessio Corti, Hiroshi Iritani, and Hsian-Hua Tseng. It introduces material which will be used in the talks by Corti, Iritani, Ruan, and Tseng.

Alessio Corti

Quantum orbifold cohomology of weak Fano toric DM stacks

I outline a recipe for computing the small quantum orbifold cohomology of a weak Fano toric DM stack X. This is completely explicit if X is Fano and has terminal singularities. In general, the recipe requires the inversion of a "mirror map". This is joint work with T Coates, H Iritani and H-H Tseng.

Rebecca Goldin

Chen-Ruan Cohomology for global quotients by abelian Lie groups

Let X be an orbifold obtained as a global quotient of a manifold by a finite group, which is not necessarily abelian. Fantechi and Gottsche devised an approach to calculate its orbifold (or Chen-Ruan) cohomology, using a larger ring with a G-action and calculating its invariants.

In recent work, Tara Holm, Allen Knutson and I developed a way to generalize these techniques in certain cases. Let M be an almost complex manifold with an action by an abelian Lie group T. We define the *inertial cohomology* of the pair (M, T). In the case that T acts with finite stabilizers, this cohomology coincides with the Fantechi-Gottsche description of Chen-Ruan cohomology. However, in the case T does not act locally freely, this is a new ring worth studying in its own right. In particular, when M is a symplectic manifold and T acts in a Hamiltonian fashion, the inertial cohomology *surjects* onto the Chen-Ruan cohomology of the orbifold $M//T$, the symplectic quotient of M by T. When M is the cotangent bundle of $\mathbb{C}^n$, the inertial cohomology of (M, T) surjects onto the hyperkahler quotient $M////T$, also called a hypertoric variety (this latter result is joint with Megumi Harada). In both the symplectic and the hyperkahler case, these surjections lead to new ways of describing Chen-Ruan cohomology for the orbifolds obtained as quotients. In this talk, we will present the construction of the inertial cohomology of (M, T) including its ring structure, show how it is related to the work of Fantechi and Gottsche, and (briefly) discuss these surjectivity theorems.

Megumi Harada

The K-theory of symplectic quotients

Let G be a compact connected Lie group, and (M, ω) a Hamiltonian G-space with proper moment map μ. A classical theorem of Kirwan states that there is a surjective ring map κ from the G-equivariant rational cohomology of M surjects onto the ordinary rational cohomology ring of the symplectic quotient $M//G$. The Kirwan surjectivity theorem, in addition to computations of the kernel of κ, give powerful methods for explicit computations of the cohomology rings of symplectic quotients.

We present integral K-theoretic analogues of this theory which therefore gives methods for computing the integral K-theory of symplectic quotients. More specifically: (1) we prove a K-theoretic Kirwan surjectivity theorem; (2) give a relationship between the kernel of the Kirwan map κ_G for a nonabelian Lie group and the kernel of the Kirwan map κ_T for its maximal torus (thus allowing us to reduce computations to the abelian case; and (3) in the abelian case, give methods for explicit computations of the kernel of κ_T. Our results are K-theoretic analogues of the rational-cohomological theory developed by Kirwan, Martin, Tolman-Weitsman, and others.

Tara Holm

Symplectic techniques for computing the cohomology of orbifolds

We present techniques for computing the various cohomology rings associated to an orbifold X that arises as a symplectic quotient $M//G$. Building on the themes developed in Goldin's talk, we first define an alternative version of the stringy product on the inertial cohomology of a Hamiltonian T-space. This new product avoids mention of an obstruction bundle, is clearly associative, and is combinatorial in nature. We use this and surjectivity results to give a combinatorial description of the Chen-Ruan coho-

mology of an abelian symplectic reduction. We conclude with several examples, and a brief discussion of coefficient rings. This talk is based on joint projects with Goldin and Knutson; Sjamaar; and Tolman.

Hiroshi Iritani

Wall-crossing in the quantum cohomology of toric orbifolds

My talk represents joint work with Tom Coates, Alessio Corti and Hsian-Hua Tseng. We study the change in quantum cohomology under a wall-crossing of Kaehler classes. Quantum cohomology is considered to be a family of rings over $H^{1,1}(X)$, the complexified Kaehler moduli space. Moreover, this family of rings comes from Hodge theory called "quantum D-module". In the toric case, using an explicit mirror family, we can construct a global "Kaehler moduli" and a global quantum D-module over it. This global moduli space has several cusps, each of which corresponds to a toric orbifold. The different orbifolds are related by wall-crossings. A neighbourhood of each cusp is the Kaehler moduli of the corresponding toric orbifold. The Fourier expansion of the global D-module connection at each cusp gives the genus zero Gromov-Witten invariants of the corresponding toric orbifold. This (modulo a mirror conjecture) establishes a quantum cohomology version of the McKay correspondence (Ruan's conjecture) in case of toric orbifolds. One interesting observation here is that whereas the family of rings is globally well-defined, the Frobenius (or flat) structure associated with the quantum cohomology varies from cusp to cusp.

Dagan Karp

Large N duality and orbifold quotients of the resolved conifold

Large N duality is a conjectural correspondence relating SU(N) Chern-Simons theory of real 3-manifolds to Gromov-Witten theory of complex 3-folds. In this talk I'll discuss CS/GW duality for certain lens spaces, whose duals are (resolutions of) cyclic quotients of the resolved conifold. Removing the parentheses in the previous sentence necessitates orbifold Gromov-Witten theory. The (resolved) lens space duals are toric Calabi-Yau threefolds; hence the method on the GW side consists of the topological vertex. This confirms a conjecture of Aganagic-Klemm-Marino-Vafa posed in 2002, and is joint work with Dave Auckly and Sergiy Koshkin.

Ralph Kaufmann

The global orbifold approach to stringy geometry

As has been demonstrated in many constructions such as orbifold cohomology by Fantechi and Goettsche or in orbifold K-theory by T. Jarvis, T. Kimura and myself, there are certain richer structures for an orbifold which admits a representation as a global quotient. We discuss old and new stringy phenomena from this global orbifold perspective.

Takashi Kimura

Stringy Algebraic Structures and Orbifolds

Associated to a smooth, projective variety with the action of a finite group G is its stringy cohomology ring, a G-Frobenius algebra introduced by Fantechi-Goettsche whose G-coinvariants yield the Chen-Ruan orbifold cohomology of the quotient orbifold. We contrast the different definitions of this structure and explain the role of the moduli space of pointed admissible G-covers. We will also discuss some questions and generalizations of these constructions.

Etienne Mann

Orbifold quantum cohomology of weighted projective spaces

In 2001, S. Barannikov showed that the Frobenius manifold coming from the quantum cohomology of the complex projective space of dimension n is isomorphic to the Frobenius manifold associated to the Laurent polynomial $x_1 + ... + x_n + (x_1...x_n)^{-1}$ We propose to explain how we can extend this correspondence to weighted projective space and a certain Laurent polynomial.

Fabio Perroni

Chen-Ruan cohomology of ADE singularities

We study Ruan's "cohomological crepant resolution conjecture" for orbifolds with transversal ADE singularities. In the A_n-case we compute both the

Chen-Ruan cohomology ring $H^*_{\mathrm{CR}}([Y])$ and the quantum corrected cohomology ring $H^*(Z)(q_1, ..., q_n)$. The former is achieved in general, the later up to some additional, technical assumptions. We construct an explicit isomorphism between $H^*_{\mathrm{CR}}([Y])$ and $H^*(Z)(-1)$ in the A_1-case, verifying Ruan's conjecture. In the A_n-case, the family $H^*(Z)(q_1, ..., q_n)$ is not defined for $q_1 = ... = q_n = -1$ as the conjecture would require. We propose a slightly modified conjecture in the A_n-case which we prove in the A_2-case, and for $P(1, 3, 4, 4)$, by constructing explicit isomorphisms.

Yongbin Ruan
Quantum birational geometry

In the early day of quantum cohomology era, there was an interesting question called "naturality of quantum cohomology". Namely, what are the "morphism"s between symplectic/projective manifolds such that they induces a homomorphism on quantum cohomology. It was quickly realized that this is a very deep question and related to birational geometry. In the mid-90's, a set of conjectures were made to pin down these relationships. Very little progress was made on these conjectures over the last ten years, partially due to the fact that there are still some pieces of information missing from the general formulation of the conjecture. With the advance in orbifold theory, I will explain that the B-field and the symplectic transformation are precisely the missing ingredients.

Michael Rose
Arithmetic mirror symmetry and ℓ-adic Chen-Ruan cohomology

The Weil conjectures provide a technique to translate certain mirror theorems into the context of arithmetic algebraic geometry. I will demonstrate this strategy and give a survey of results in this direction.

Hsian-Hua Tseng
Twisted Orbifold Gromov-Witten Invariants in Genus Zero

In this talk we will discuss an explicit procedure relating genus-zero twisted orbifold Gromov-Witten invariants to usual orbifold Gromov-Witten invariants. We will also discuss its application to the calculation of genus zero orbifold Gromov-Witten invariants of $\mathbb{C}^3/\mathbb{Z}_3$. This is joint work with Tom Coates, Alessio Corti, and Hiroshi Iritani.

Angelo Vistoli
Essential dimension and algebraic stacks

I will report on joint work with Patrick Brosnan and Zinovy Reichstein. We extend the notion of "essential dimension" that has been studied so far for algebraic group, to algebraic stacks. The problem is the following: given a geometric object X over a field K (e.g., an algebraic variety), what is the least transcendence degree of a field of definition of X over the prime field? In other words, how many independent parameters do we need to define X? We have complete results for smooth, or stable, curves. Furthermore the stack-theoretic machinery that we develop can also be applied to the case of case of algebraic groups, showing for example that the essential dimension of $Spin_n$ grows exponentially with n.