WAGS

The Western Algebraic Geometry Symposium

Vancouver, May 1–2, 2010

Saturday, May 1			
9:00	Lounge	Registration, Breakfast	
10:00	1100	**Mark Gross**	Smoothing cusp singularities via mirror symmetry
11:30	1100	**Massimilliano Mella**	On fiber type morphisms of $\overline{M}_{0,n}$
12:30	Lounge	Lunch	
2:00	1100	**Joel Kamnitzer**	Categorical Lie algebra actions and equivalence of derived categories of coherent sheaves
3:00	Lounge	**Poster Session**	
3:30	1100	**Amin Gholampour**	Counting invariants for the ADE McKay quivers
5:30	TBA	Reception	

Sunday, May 2			
9:00	Lounge	Breakfast	
9:30	1100	**Aravind Asok**	Connectedness, homotopy theory and birational invariants
11:00	1100	**Sam Payne**	Tropical Brill-Noether theory

Abstracts

Mark Gross
Smoothing cusp singularities via mirror symmetry

I will talk about joint work with Paul Hacking and Sean Keel. We use mirror symmetry to prove a conjecture of Looijenga about smoothability of cusp surface singularities. These are normal singularities whose minimal resolution has an exceptional locus given by a cycle of rational curves. It was known that these singularities come in pairs, called "dual cusps". It turns out that this is a manifestation of mirror symmetry, and using recent results of Gross-Siebert and Gross-Pandharipande-Siebert, we are able to prove Looijenga's conjecture, which states that a cusp singularity is smoothable if and only if the cycle of rational curves corresponding to the minimal resolution of the dual cusp can be realised as the anti-canonical class of a rational surface.

Massimillano Mella
On fiber type morphisms of $\overline{M}_{0,n}$

The geometry of $\overline{M}_{0,n}$ is frequently controlled by combinatorial arguments. In this talk I will try to show a different approach, naturally inherited by Kapranov's beautiful construction of this moduli space as blow up of the projective space.

The main question I will confront with is the following: Is any fiber type morphism from $\overline{M}_{0,n}$ factored via a forgetful map?

Using projective techniques partial interesting answer will be proposed.

Joel Kamnitzer

Categorical Lie algebra actions and equivalence of derived categories of coherent sheaves.

Actions of the Lie algebra $\mathfrak{sl}(2)$ on vector spaces arise naturally in combinatorics, geometry, and algebra. Such an action consists of a sequence of vector spaces with linear maps between them satisfying certain relations. From this perspective, one can define an action of $\mathfrak{sl}(2)$ on a category to be a sequence of categories with functors between them satisfying certain relations. Such actions were studied by Chuang-Rouquier in the context of representations of the symmetric group in positive characteristic. More recently, Cautis, Licata, and the speaker studied an action of $\mathfrak{sl}(2)$ where the categories involved were derived categories of coherent sheaves on cotangent bundles to Grassmannians. Following the ideas of Chuang-Rouquier, we used this $\mathfrak{sl}(2)$ action to construct an equivalence of derived categories between different cotangent bundles of Grassmannians. We also generalized this construction to give categorical Kac-Moody Lie algebra actions on derived categories of quiver varieties. In this setting the equivalences lead to braid group actions.

Amin Gholampour

Counting invariants for the ADE McKay quivers

We study the moduli space of the McKay quiver representations Q associated to the finite subgroups $G < SU(3)$. Let $p : Y = G\text{-Hilb}(C^3) \to \mathbb{C}^3/G$ be the natural Calabi-Yau resolution. In the cases where the fibers of p are at most 1 dimensional, there is an equivalence of the abelian categories of such representations and the perverse sheaves on Y relative to p. By defining certain stability conditions on these abelian categories, the moduli spaces of Donaldson-Thomas and Pandharipande-Thomas invariants on Y, and of Szendroi invariants on Q are recovered. In the special case where $G < SU(2) < SU(3)$, we prove a relation between these invariants by means of the wall crossing. This gives explicit formulas for the invariants which allows verifying the conjectural Gromov-Witten/Donaldson-Thomas correspondence for Y, and the Donaldson-Thomas Crepant Resolution Conjecture for p.

Aravind Asok

Connectedness, homotopy theory and birational invariants

The classical Luroth problem asks whether every subfield of a purely transcendental extension of a field k is itself purely transcendental. Much work was done in the 1970s to produce counterexamples to the Luroth problem. I will examine one approach (that of Artin and Mumford, as generalized by Colliot-Thelene and Ojanguren) that uses cohomological invariants to detect counterexamples. More precisely, I will discuss how techniques from the Morel-Voevodsky homotopy theory of algebraic varieties allow one to use 'higher' cohomological invariants to produce new counterexamples to the Luroth problem over the complex numbers.

Sam Payne

Tropical Brill-Noether theory

Classical Brill-Noether theory studies the geometry of special divisors on smooth projective curves, with special attention to the existence of special divisors on the general curve of genus g. The fundamental result of the theory, called the Brill-Noether Theorem and due to Griffiths and Harris, says that a general curve of genus g has a divisor of degree d that moves in a linear system of dimension r if and only if g is at least $(r+1)(g-d+r)$. The original proof uses a degeneration to a rational curve with g nodes, plus a very subtle transversality argument for certain associated Schubert cycles.

I will present a new and simpler proof of the Brill-Noether Theorem, using a different degeneration, to a union of smooth rational curves whose dual graph is encoded in a tropical curve with first Betti number g. The proof relies heavily on the theory of divisors on graphs pioneered by Baker and Norine, and gives an explicit criterion for a curve to be Brill-Noether general over a discretely valued field, such as $\mathbb{Q}$. This is joint work with Filip Cools, Jan Draisma, and Elina Robeva.