WAGS

The Western Algebraic Geometry Seminar

Fall 2003, Vancouver

Room 1100, Mathematics Annex, University of British Columbia

Saturday, September 13, 2003		
9:00	Coffee	
9:30	**Sándor Kovács**	Recent advances in the Minimal Model Program,
11:00	**James McKernan**	after Shokurov, I–II
12:00	Lunch	
1:30	**Dan Abramovich**	Valuative criteria for stable complexes
2:30	Coffee	
3:00	**Alexander Polishchuk**	A-infinity homogeneous coordinate rings
4:30	**Gavril Farkas**	The Mori cones of moduli spaces of pointed curves
6:00	Barbecue	

Sunday, September 14, 2003		
9:00	Coffee	
9:30	**Bernd Sturmfels**	Tropical Algebraic Geometry
11:00	**Ravi Vakil**	A geometric Littlewood-Richardson rule
1:00	informal Dim Sum	

Abstracts

Sándor Kovács, James McKernan
Recent advances in the Minimal Model Program, after Shokurov, I-II

One of the major discoveries of the last two decades in algebraic geometry is the realization that the theory of minimal models of surfaces can be generalized to higher dimensional varieties. The major initial architects of the resulting theory in the 1980s were Y. Kawamata, J. Kollár, S. Mori, M. Reid, and V. V. Shokurov. They have built a theory of minimal models that works in all dimensions except for one crucial step: the existence and termination of flips.

Flips are birational operations that only appear in higher dimensions and their definition does not assure their existence. Nevertheless they are essential to obtaining minimal models. It has proved extremely difficult to show the existence of flips. Mori proved their existence in dimension three, which earned him the Fields Medal in 1990, but there has been very little advance in dimensions four and higher for a long time.

Recently Shokurov introduced revolutionary new ideas that immediately gave a more theoretical proof of the three-dimensional case and may lead to a complete solution to the problem.

In the first talk, the Minimal Model Program will be introduced, including key definitions, theorems, and procedures. Flips will be defined and their importance discussed.

In the second talk, Shokurov's new ideas will be discussed and put into perspective with regard to the previous ideas of the theory.

Dan Abramovich

Valuative criteria for stable complexes

Motivated by ideas from Physics, Tom Bridgeland defined a notion of a stability condition on the derived category of coherent sheaves on a projective manifold. This notion generalizes the notion of stability of vector bundles on curves. Bridgeland went on to prove that the collection of all stability conditions on the derived category of a projective manifold forms a complex manifold - one which is of much interest for mathematical physicists. A different natural mathematical question is the study of moduli spaces of stable objects under a given stability condition.

I will review Bridgeland's stability conditions, and discuss joint work with Alexander Polishchuk proving valuative criteria for properness and separation for the moduli of semistable objects under a noetherian stability condition.

Alexander Polishchuk

A_∞ homogeneous coordinate rings

The study of A_∞-structures on the derived categories of coherent sheaves on projective varieties is one of the facets of the mirror symmetry phenomenon. The main topic of this talk will be the theorem stating that in many situations these A_∞-structures are essentially unique.

Gavril Farkas

The Mori cones of moduli spaces of pointed curves

We discuss a conjectural description of the Mori cone of the moduli space of n-pointed stable curves of genus g and how this is related to a conjecture of Fulton's on the cone of effective divisors on the moduli space of n-pointed rational curves. We then present an entirely combinatorial approach to Fulton's conjecture and explain how this works in the case of low dimensional moduli spaces.

Bernd Sturmfels

Tropical Algebraic Geometry

In tropical algebraic geometry, the zero sets of polynomial equations are piecewise-linear. The tropical variety of a polynomial ideal is the 'logarithmic limit set' which was introduced by Bergman in 1971 and studied further by Bieri and Groves in the 1980's. We show that it is embedded as a polyhedral subcomplex in the Gröbner fan of the ideal. Our main example is the Grassmannian of lines in tropical projective space. We interpret tropical lines as phylogenetic trees, and we identify the Grassmannian with the space of trees of Billera, Holmes and Vogtmann. The references are math.AG/0306366 and math.AG/0304218 .

Ravi Vakil

A geometric Littlewood-Richardson rule

I will describe an explicit geometric Littlewood-Richardson rule, interpreted as deforming the intersection of two Schubert varieties so that they break into Schubert varieties. There are no restrictions on the base field, and all multiplicities arising are 1; this is important for applications. This rule should be seen as a generalization of Pieri's rule to arbitrary Schubert classes, by way of explicit homotopies. It has a straightforward bijection to other Littlewood-Richardson rules, such as tableaux and Knutson and Tao's puzzles.

This gives the first geometric proof and interpretation of the Littlewood-Richardson rule. It has a host of geometric consequences, which I may describe, time permitting. The rule also has an interpretation in K-theory, suggested by Buch, which gives an extension of puzzles to K-theory, and in fact a Littlewood-Richardson rule in equivariant K-theory (ongoing work with Knutson). The rule suggests a natural approach to the open question of finding a Littlewood-Richardson rule for the flag variety, leading to a conjecture, shown to be true up to dimension 5. Finally, the rule suggests approaches to similar open problems, such as Littlewood-Richardson rules for the symplectic Grassmannian and two-flag varieties.