Correction to: Loop Erased Walks and Uniform Spanning Trees.

M. T. Barlow*

January 25, 2017

Theorem 5.2, which is stated as being ‘implicit in [BJKS, KM08], is actually false. The bounds on the resistances $R_{\text{eff}}(o, x)$ are not enough to give control of the quantity that is needed, which is $R_{\text{eff}}(o, B(o, R)^c)$. Fortunately Theorem 5.2 was given for illustration only; the result which is used in the paper is Theorem 5.3, which is proved in [BJKS, KM08].

For Theorem 5.1 the control on $R_{\text{eff}}(o, B(o, R)^c)$ is obtained from the point to point resistances by using estimates on $V(x, r)$ and $R_{\text{eff}}(x, y)$ for base points $x$ other than just $o$.

To see that Theorem 5.2 is actually false, rather than just not proved, consider the following example. Let $G$ be a ‘comb graph’ of the following type. We take $G = (V, E)$ where $V = \mathbb{Z}_+^2$ and the the edges are of the form

$$E = \{(k, 0), (k + 1, 0), k \geq 0\} \cup \{(k, j), (k, j + 1), k \geq 0, j \geq 0\}.$$

Then $G$ is a tree, and if $o = (0, 0)$ we have $V(o, r) \asymp r^2$, so this graph satisfies the conditions of Theorem 5.2 with $\alpha = 2$.

Let $r \geq 1$, and $k = r^{1/2}$. Consider the flow $I$ from $o$ to $B(o, r)^c$ obtained by making a flow of $1/k$ upwards in each of the first $k$ teeth. This flow has energy $E(I)$ bounded by

$$\sum_{i=0}^{k} (r - i)k^{-2} + \sum_{i=0}^{k} (1 - i/k) \leq rk^{-1} + k \leq 2r^{1/2}.$$

Thus $R_{\text{eff}}(o, B(o, r)^c) \leq cr^{1/2}$.

Hence, writing $B = B(o, r)$,

$$\mathbb{E}^o \tau(o, r) \leq \sum_{x \in B} g_B(o, x) \leq g_B(o, o)V(o, r) \leq cr^{5/2} = cr^{\alpha + 1/2}.$$

I am grateful to Tom Hutchcroft for pointing me to this error.

*Research partially supported by NSERC (Canada)
References


Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Z2, Canada.
barlow@math.ubc.ca