420/507 Homework 1
Due Monday 18 September 2017

1. Folland p. 24 Qu. 3.

2. Folland p. 24 Qu. 4.

3. Let \(X, Y\) be (non-empty) sets and \(f : X \to Y\). Suppose that \(\mathcal{F}\) is a \(\sigma\)-algebra on \(X\), and \(\mathcal{G}\) is a \(\sigma\)-algebra on \(Y\).
   (a) Prove that \(\{f^{-1}(G) : G \in \mathcal{G}\}\) is a \(\sigma\)-algebra on \(X\).
   (b) Is \(\{f(F) : F \in \mathcal{F}\}\) a \(\sigma\)-algebra on \(Y\)? If true give a proof, if false a counterexample.

4. Let \(\mathcal{E} = \{F \subset \mathbb{R} \text{ such that } F \text{ is finite}\}\). What is \(\mathcal{M}(\mathcal{E})\), the \(\sigma\)-algebra generated by \(\mathcal{E}\)?

5. Let \(\mathcal{E}_1 = \{(p, q] : p, q \in \mathbb{Q}, p < q\}\). What is \(\mathcal{M}(\mathcal{E}_1)\)?

6. Let \((X, d)\) be a non empty metric space, and \(\mathcal{O}\) be the collection of open sets. When is \(\mathcal{O}\) a \(\sigma\)-algebra?