1. (a) Give an example to show that if $F$ and $G$ are $\sigma$-fields of subsets of $\Omega$ then $F \cup G$ need not be a $\sigma$-field.
(b) Let $(\Omega_i, F_i), i = 1, 2$ be measure spaces. Show that in general
\[ \mathcal{E} = \{F_1 \times F_2 : F_i \in F_i\} \]
is not a $\sigma$-field.


6. Let $\Omega = \mathbb{N} = \{1, 2, \ldots\}$, and
\[ A = \{A \subset \Omega : A \text{ is finite}\} \]
\[ B = \{B \subset \Omega : B^c \text{ is finite}\} \]
\[ \mathcal{E} = A \cup B. \]

(a) Show that $\mathcal{E}$ is a field but not a $\sigma$-field.
(b) Let $P : \mathcal{E} \to [0,1]$ be defined by
\[ P(A) = \begin{cases} 0, & \text{if } A \in A, \\ 1, & \text{if } A \in B. \end{cases} \]
Prove that $P$ is finitely additive, but not countably additive.