Show all work clearly and in order, and circle your final answers.

(5 Points) Draw the level curves of $z = f(x, y)$ and $z = g(x, y)$ at $z = 0, 1$ and 2:

- $f(x, y) = \sqrt{x^2 + y^2 - 1}$
- $g(x, y) = \sqrt{1 + x^2 + y^2}$

Of the four graphs pictured below, identify which is a graph of $f(x, y)$ and which is a graph of $g(x, y)$. The tick marks are at 1, 2, 3 and 4 on the axes.
(2 Points) Compute $f_{xx}$, $f_{xy}$, and $f_{yy}$ for $f(x, y) = e^{xy^2} - y - x^2$.

$$
\partial_x = y^2 e^{xy^2} - 2x, \quad \partial_y = 2xy e^{xy^2} - 1 \\
\partial_{xx} = y^4 e^{xy^2} - 2 \\
\partial_{xy} = 2x e^{xy^2} + x^2 y^2 e^{xy^2} \\
\partial_{yy} = 2y e^{xy^2} + 2xy^3 e^{xy^2}
$$

(3 Points) Use the total differential of $f(x, y) = \ln(x - y)$ at $(5, 4)$ to estimate $f(5.1, 3.98)$.

$$
\Delta x = \frac{1}{x - y}, \quad \Delta y = \frac{-1}{x - y} \\
\Delta x (5, 4) = 1, \quad \Delta y (5, 4) = -1
$$

$$
\approx f(5, 4) (\Delta x) (x - 5) + f_y (5, 4) (\Delta y) (y - 4) + f(5, 4)
$$

$$
x \approx 5.02 \\
y \approx 3.98
$$

(Bonus) Does there exist a function $f(x, y)$ such that $f_x = \frac{y}{x^2 + y^2}$ and $f_y = -\frac{x}{x^2 + y^2}$? If so, find $f(x, y)$ and if not justify your answer.

No, + reasonable explanation, like if $f$ existed

$$
f(x, y) = \arctan \left( \frac{x}{y} \right)
$$

But

$$
\partial_y = -\frac{x}{x^2 + y^2} = \frac{1}{y^2 + x^2} \frac{y}{y^2} + \partial_y (y) , \quad \text{g depends on x,} \\
\text{so no such function g alone.}
$$