**Basic Prisoners Dilemma** Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge. They hope to get both sentenced to a year in prison on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either to: betray the other by testifying that the other committed the crime, or to cooperate with the other by remaining silent. The offer is:

- If A and B each betray the other, each of them serves 2 years in prison
- If A betrays B but B remains silent, A will be set free and B will serve 3 years in prison (and vice versa)
- If A and B both remain silent, both of them will only serve 1 year in prison (on the lesser charge)

What should you do?

**Iterated Prisoners Dilemma** Assume that there are $N$ charges and between each charge both prisoners hear what the other prisoner plead and may update their plea accordingly. A simple model for this is a finite state model with four states, where each prisoner tries to decide what to do on the $n + 1$’st charge based only on what the other prisoner did for the $n$’th charge. The states are then

1. **SS** - Both stay silent
2. **SB** - Prisoner 1 stays silent, prisoner 2 betrays
3. **BS** - Prisoner 1 betrays, prisoner 2 stays silent
4. **BB** - Both betray

**Question 1** Draw the state graph for this model.

Now, fix the following probabilities:

1. $S_1$ - Probability that **Prisoner 1** chooses to stay silent IF **Prisoner 2** did last time (regardless of what prisoner 1 choose last time)
2. $B_1$ - Probability that **Prisoner 1** chooses to betray IF **Prisoner 2** did last time (regardless of what prisoner 1 choose last time)
3. $S_2$ - Probability that **Prisoner 2** chooses to stay silent IF **Prisoner 1** did last time (regardless of what prisoner 2 choose last time)
4. $B_2$ - Probability that **Prisoner 2** chooses to betray IF **Prisoner 1** did last time (regardless of what prisoner 2 choose last time)

**Question 2** Label each arrow in your graph with the transition probabilities in terms of $S_i$ and $B_i$.

**Question 3** Find the transition matrix in terms of $S_i$ and $B_i$. Hint: If $S_1 = S_2 = B_1 = B_2 = 1$ is should be

$$ T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

**Question 4**: If Prisoner 1 uses tit for tat method, $S_1 = B_1 = 1$, what strategy should Prisoner 2 play?

**Question: Research Level** What is the best strategy overall?