1. **POWER SERIES**

Power series method for solving differential equations

\[ x''(t) + 2x(t) = t^3, \]

and we write

\[ x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \ldots \]

(1)

and the equation becomes

\[ 2a_2 + 6a_3 + \ldots + 2a_0 + 2a_1 t + 2a_2 t^2 + 2a_3 t^3 + \ldots = t^3. \]

To solve this we only need to consider up to \( a_3 \) as in last class. We could deal with exponents and sines and other functions if we got very good with dealing with their Taylor expansions. This was actually the method that Euler used to solve differential equations in the 1700s! We have forgotten a lot of the art of manipulating power series but have much better techniques.

2. **FORCED SPRING**

We will work with a forced, undamped, spring equation (or circuit or linear pendulum)

\[ x''(t) + \mu^2 x(t) = f(t), \]

with the angular frequency \( \mu > 0 \). The homogeneous solution is

\[ x_H(t) = c_1 \cos(\mu t) + c_2 \sin(\mu t). \]

Interesting phenomena occurs when the forcing is periodic:

\[ f(t) = f_0 \cos(\omega t). \]
3. Workshop 5

4. Beats and Resonance

When $\mu \neq \omega$ the solutions are bounded for all time. The two frequency form lower frequency ‘beats’ when they are close. Using trig the resonant frequency is $\mu - \omega$.

\[
A \cos(\mu t) = A \cos\left(\frac{\mu + \omega}{2} t + \frac{\mu - \omega}{2} t\right)
\]
\[
= A \cos\left(\frac{\mu + \omega}{2} t\right) \cos\left(\frac{\mu - \omega}{2} t\right) - A \sin\left(\frac{\mu + \omega}{2} t\right) \sin\left(\frac{\mu - \omega}{2} t\right),
\]

\[
B \cos(\omega t) = B \cos\left(\frac{\mu + \omega}{2} t - \frac{\mu - \omega}{2} t\right)
\]
\[
= B \cos\left(\frac{\mu + \omega}{2} t\right) \cos\left(\frac{\mu - \omega}{2} t\right) - B \sin\left(\frac{\mu + \omega}{2} t\right) \sin\left(\frac{\mu - \omega}{2} t\right).
\]

Summed together, assuming they have the same amplitude, these are

\[
A \cos(\mu t) + A \cos(\omega t) = 2A \cos\left(\frac{\mu + \omega}{2} t\right) \cos\left(\frac{\mu - \omega}{2} t\right)
\]