Goal 1: Graph $f$ given information about $f'$

Goal 2: Relate $f'$ to local extrema.

**Increasing:**

![Graph showing increasing function]

**Decreasing:**

![Graph showing decreasing function]

Weakly increasing on an interval $I$: 
$f(x_2) \geq f(x_1) \quad x_2 \geq x_1, \quad x_2, x_1 \in I$

Weakly decreasing on an interval $I$: 
$f(x_2) \leq f(x_1) \quad x_2 \leq x_1, \quad x_2, x_1 \in I$
Weakly increasing:

\[ \text{Thm (Test for weakly increasing/decreasing function)} \]
Suppose \( f \) is cont. on int. \( I \) and differentiable on interior of \( I \). Then

\[ f' > 0 \Rightarrow \text{weakly increasing on } I \]
\[ f' \leq 0 \Rightarrow \text{weakly decreasing on } I \]

This applies to \( f(x) = x^3 \) on \( (-\infty, \infty) \)
(\text{an increasing function})

Note \( f'(x) = 3x^2 \)

\[ \Rightarrow f'(0) = 0, \quad f'(x) \geq 0 \quad x \in (-\infty, \infty). \]
Ex) Sketch a function satisfying:

a) $f' > 0$ on $(-\infty, 0), (4, 6), (6, \infty)$

b) $f' < 0$ on $(0, 4)$

c) $f'(6)$ is not defined

d) $f'(4) = f'(6) = 0$

e) $f$ is continuous.

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<table>
<thead>
<tr>
<th>sign of $f'$</th>
<th>+</th>
<th>-</th>
<th>+</th>
<th>+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Behaviour of $f$</td>
<td>increasing</td>
<td>dec.</td>
<td>inc.</td>
<td>inc.</td>
</tr>
<tr>
<td>$f$</td>
<td>$-\infty$</td>
<td>corner or cusp</td>
<td>4 flat</td>
<td>6 flat</td>
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</tbody>
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Graph of $f$:  

- corner
- flat
- flat

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(3)
Ex) Find intervals on which $xe^{-x}$ is increasing/decreasing.

Find sign of $f'$:

$$f'(x) = x(e^{-x}) + e^{-x} = (1-x)e^{-x}$$

always changes sign positive around $x=1$

$f'(x) > 0$ for $x < 1$

$f'(x) < 0$ for $x > 1$

Thus $f$ is increasing for $x < 1$

decreasing for $x > 1$

Rough sketch of graph:

<table>
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<td>Behaviour of $f$</td>
<td>inc.</td>
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$x=1$

$f'(1)=0$
$f$ is flat
Ex) Suppose $f'(x) = 12(x+1)(x-1)^2$ where $f$ is a continous function. Identify local extrema:

<table>
<thead>
<tr>
<th>sign($f'$)</th>
<th>$-$</th>
<th>$+$</th>
<th>$+$</th>
</tr>
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<tr>
<td>$-1$</td>
<td></td>
<td>$f'(1)=0$</td>
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$(x-1)^2$ is always $\geq 0$

$y = x+1$

Extrema:

local min at $x = -1$