Final 2012:

Consider the curve $x^2 + y^3 - 2xy = 0$.

Find $f''(1)$ assuming that $y = f(x)$ near $(x,y) = (1,1)$.

Solution: 1st Diff. once:

$$2x + 3y^2 y' - 2(y + xy') = 0$$

2nd Diff. again:

$$2 + 3((xyy')y' + y^2y'') - 2y - 2(y' + xy'') = 0$$

3) Plug $(1,1)$ into first eq:

$$2 + 3 \cdot y'(1) - 2(1 + 1 \cdot y'(1)) = 0$$

$$2 + y'(1) - 2 = 0$$

$$y'(1) = 0$$

4) Plug $x = 1$, $y = 1$, $y' = 0$ into sec. eq:

$$2 + 3(2 \cdot 1 - 0 + 1^2 y'') - 2y - 2(0 + y'') = 0$$

$$y'' = 0$$