### Very short answer questions

1. (a) **3 marks** What is the worth, after 9 months, of an investment of $200 with a nominal interest rate of 15% compounded quarterly?

   **Answer:** \(200 \cdot (1.0375)^{3}\)

   **Solution:** We use the compounded interest formula
   
   \[FV = PV \cdot \left(1 + \frac{i}{n}\right)^{nt} \]

   We plug in
   
   \[PV = 200, \quad n = 4, \quad t = \frac{3}{4}, \quad i = 0.15\]

   to get
   
   \[FV = 200 \cdot \left(1 + \frac{0.15}{4}\right)^{4 \cdot \frac{3}{4}} = 200 \cdot (1.0375)^{3}.\]

   (b) **2 marks** How fast is the investment (from part (a)) growing after 9 months?

   **Answer:** \(800 \cdot (1.0375)^{3} \cdot \log(1.0375)\)

   **Solution:** We have
   
   \[FV(t) = 200 \cdot \left(1 + \frac{0.15}{4}\right)^{4t} \]

   Deriving it we get
   
   \[FV'(t) = 200 \cdot \left(1 + \frac{0.15}{4}\right)^{4t} \cdot 4 \cdot \log\left(1 + \frac{0.15}{4}\right).\]

   And by plugging in \(t = \frac{3}{4}\) we find that the rate in which the investment was growing after 9 months was
   
   \[FV'\left(\frac{3}{4}\right) = 200 \cdot \left(1 + \frac{0.15}{4}\right)^{4 \cdot \frac{3}{4}} \cdot 4 \cdot \log\left(1 + \frac{0.15}{4}\right) = 800 \cdot (1.0375)^{3} \cdot \log(1.0375).\]

### Long answer questions - You must show your work

2. **7 marks** Find the absolute maximum and minimum of \(f(x) = x^{2/3} + x\) on \([-1, 1]\) (value and point).

   **Answer:** Max 2, Min 0
Solution: We derive the function
\[ f'(x) = \frac{2}{3x^{1/3}} + 1, \]
this is true for any \( x \neq 0 \). Solving the equation \( f'(c) = 0 \) for \( c \) yields one solution \( c = -\left(\frac{2}{3}\right)^3 \) which is in the interval. We have two critical points 0 and \(-\left(\frac{2}{3}\right)^3\). We plug the edges and critical points into the function
\[ f(-1) = 0, \quad f(0) = 0, \quad f\left(-\left(\frac{2}{3}\right)^3\right) = \frac{4}{27}, \quad f(1) = 2 \]
The absolute maximum is 2 at 1 and the absolute minimum is 0 at \(-1\) and 0.

3. **8 marks** A kite is flying 40 meters above the ground when the wind starts to blow it away in a direction parallel to the ground at the rate of \( 4 \text{ m} \text{ sec}^{-1} \). At what rate must the string be let out when the length of string already let out is 80 meters?

Answer: \( \frac{\sqrt{4800}}{20} \text{ m} \text{ sec}^{-1} \)

Solution: We denote the horizontal distance between the kite and the person operating it by \( x(t) \) (measured in meters) and the actual distance by \( s(t) \). We have \( s(t_0) = 80 \). We already know that \( \frac{ds}{dt}(t_0) = 4 \text{ m} \text{ sec}^{-1} \).

By the Pythagorean theorem we have \( x(t)^2 + 40^2 = s(t)^2 \). Differentiating this with respect to \( t \) yields \( 2x(t)\frac{dx}{dt} = 2s(t)\frac{ds}{dt} \). We note that \( x(t_0)^2 + 40^2 = s(t_0)^2 = 100^2 \) and hence \( x(t_0) = \sqrt{80^2 - 40^2} = \sqrt{4800} \). Plugging everything into \( 2x(t)\frac{dx}{dt} = 2s(t)\frac{ds}{dt} \) yields \( \frac{ds}{dt}(t_0) = \frac{x(t_0)}{s(t_0)} \frac{dx}{dt}(t_0) = \frac{\sqrt{4800}}{80} \frac{4}{20} = \frac{\sqrt{4800}}{20} \text{ m} \text{ sec}^{-1} \).