Very short answer questions

1. (a) **3 marks** What is the worth, after 9 months, of an investment of $250 with a nominal interest rate of 13\% compounded quarterly?

   Answer: 250 \cdot (1.0325)^3

   **Solution:** We use the compounded interest formula
   \[ FV = PV \cdot \left(1 + \frac{i}{n}\right)^{nt}. \]

   We plug in
   \[ PV = 250, \quad n = 4, \quad t = \frac{3}{4}, \quad i = 0.13 \]
   to get
   \[ FV = 250 \cdot \left(1 + \frac{0.13}{4}\right)^{4 \cdot \frac{3}{4}} = 250 \cdot (1.0325)^3. \]

   (b) **2 marks** How fast is the investment (from part (a)) growing after 9 months?

   Answer: 1,000 \cdot (1.0325)^3 \cdot \log (1.0325)

   **Solution:** We have
   \[ FV(t) = 250 \cdot \left(1 + \frac{0.13}{4}\right)^{4t} \]

   Deriving it we get
   \[ FV'(t) = 250 \cdot \left(1 + \frac{0.13}{4}\right)^{4t} \cdot 4 \cdot \log \left(1 + \frac{0.13}{4}\right). \]

   And by plugging in \( t = \frac{3}{4} \) we find that the rate in which the investment was growing after 9 months was
   \[ FV' \left(\frac{3}{4}\right) = 250 \cdot \left(1 + \frac{0.13}{4}\right)^{4 \cdot \frac{3}{4}} \cdot 4 \cdot \log \left(1 + \frac{0.13}{4}\right) = 1,000 \cdot (1.0325)^3 \cdot \log (1.0325). \]

Long answer questions - You must show your work

2. **7 marks** Find the absolute maximum and minimum of \( f(x) = x^{2/3} + x \) on \([-1, 1]\) (value and point).

   Answer: Max 2, Min 0
Solution: We derive the function

\[ f'(x) = \frac{2}{3x^{1/3}} + 1, \]

this is true for any \( x \neq 0 \). Solving the equation \( f'(c) = 0 \) for \( c \) yields one solution \( c = -\left(\frac{2}{3}\right)^3 \) which is in the interval. We have two critical points 0 and \( -\left(\frac{2}{3}\right)^3 \). We plug the edges and critical points into the function

\[ f(-1) = 0, \quad f(0) = 0, \quad f\left(-\left(\frac{2}{3}\right)^3\right) = \frac{4}{27}, \quad f(1) = 2 \]

The absolute maximum is 2 at 1 and the absolute minimum is 0 at \(-1\) and 0.

3. **8 marks** A kite is flying 30 meters above the ground when the wind starts to blow it away in a direction parallel to the ground at the rate of \( 3 \text{ m/sec} \). At what rate must the string be let out when the length of string already let out is 60 meters?

**Answer:** \( \frac{\sqrt{2700}}{20} \text{ m/sec} \)

Solution: We denote the horizontal distance between the kite and the person operating it by \( x(t) \) (measured in meters) and the actual distance by \( s(t) \). We have

\[ \text{We look for } \frac{ds}{dt}(t_0), \]

where \( s(t_0) = 60 \). We already know that \( \frac{dx}{dt} = 3 \text{ m/sec} \).

By the Pythagorean theorem we have \( x(t)^2 + 30^2 = s(t)^2 \). Differentiating this with respect to \( t \) yields \( 2x(t) \frac{dx}{dt} = 2s(t) \frac{ds}{dt} \). We note that \( x(t_0)^2 + 30^2 = s(t_0)^2 = 60^2 \) and hence \( x(t_0) = \sqrt{60^2 - 30^2} = \sqrt{2700} \). Plugging everything into \( 2x(t) \frac{dx}{dt} = 2s(t) \frac{ds}{dt} \) yields \( \frac{ds}{dt}(t_0) = \frac{x(t_0) \frac{dx}{dt}}{s(t_0) \frac{ds}{dt}(t_0)} = \frac{\sqrt{2700}}{60} \cdot 3 = \frac{\sqrt{2700}}{20} \text{ m/sec} \).