**Introductory Example - How to price your merchandise?**

Let's define a few terms:

- The **revenue** is the amount of money \( R \) that a company receives by selling \( q \) items at a set price \( p \).

\[
R = p \cdot q
\]

- The point \((p,q)\) in the plane is called a **data point**.

- The **cost** is the amount of money \( C \) a company spends to make \( q \) items.

\[
C(q) = F + V(q)
\]

  - \( F \) stands for the fixed costs: Salaries, rent, commercials, etc.
  - \( V(q) \) stands for the variable costs: Materials, over-time, etc.

- The **profit** is the amount of money \( P \) the company is left with after all products were sold and all costs are paid.

\[
P = R - C
\]

- **Demand** is the relation between the price \( p \) of an item and the quantity \( q \) of items to be sold at that price. A basic principle of economy is that an increase in price leads to a decrease in demand.

**Today:** We assume that the demand is a linear function, i.e.

\[
q = A \cdot p + B
\]

What is the **main** goal of a good business?

1. Maximize revenue.
2. Minimize cost.
3. Maximize profit.
The story:
We were hired by BChalk Inc. They are selling a chalk box for $2 and sell 3,000 boxes a month.
Last April, they had a chalk sale (the chalk-fest) and, at a discount of 10 cents a box, they sold 100 more boxes than other months.

Talking with BChalk's accountant we found that their fixed cost is $3,250 a month and it costs an extra 75 cents to make a box of chalk, so now their monthly profit is $500 and they would wish to increase it.

1. Find the linear demand equation for a box of chalk. Use the notation \( p \) for the unit price and \( q \) for the weekly demand.
2. Find the monthly cost function, \( C=C(q) \), for producing \( q \) boxes of chalk per month. Note that \( C(q) \) is a linear function.
3. Find the monthly revenue function, \( R=R(q) \). Note that \( R(q) \) is a quadratic function.
4. The break-even points are where Cost equals Revenue; that is, where \( C(q)=R(q) \). Find the break-even points for the product.
5. On the same set of axes, sketch graphs of \( C=C(q) \) and \( R=R(q) \) and use these graphs to help you explain why there are two break-even points.
6. Find the profit function \( P(q)=R(q)-C(q) \). Note that it is a quadratic function.
7. Graph \( P=P(q) \) on the same axes as you sketched the graphs of \( C(q) \) and \( R(q) \). On this graph, indicate the regions of profit \( (P(q)>0) \) and loss \( (P(q)<0) \).
8. How should BChalk Inc. operate in order to maximize the weekly profit \( P=P(q) \)? Use mathematics in your explanation.