Short answer questions — you must show your work

1. 2 marks An investment of $14,000 compounded quarterly with nominal interest rate of 5% gained $2,000. What was the duration of the investment?

Answer: \( T = \frac{\ln(\frac{8}{7})}{4 \ln(1 + 0.05/4)} \) years

Solution: We use compounded interest

\[
FV = PV \cdot \left(1 + \frac{i}{N}\right)^{NT}
\]

Where \( PV = 14,000 \), \( FV = 14,000 + 2,000 = 16,000 \), \( N = 4 \) and \( i = 0.05 \). Plugging these yields

\[
16,000 = 14,000 \cdot \left(1 + \frac{0.05}{4}\right)^{4T}
\]

and hence

\[
\frac{16}{14} = \left(1 + \frac{0.05}{4}\right)^{4T}
\]

Taking ln of both sides yields

\[
\ln \left(\frac{8}{7}\right) = \ln \left[ \left(1 + \frac{0.05}{4}\right)^{4T} \right] = 4T \ln \left(1 + \frac{0.05}{4}\right).
\]

It follows that

\[
T = \frac{\ln(\frac{8}{7})}{4 \ln(1 + 0.05/4)}.
\]

(which is a little more than 2.5 years)

2. 2 marks Evaluate

\[
\lim_{{x \to 2}} \frac{3x^2 - 5x - 2}{x^2 - 5x + 6}
\]

Answer: \(-7\)
Solution: We first check that we can’t just plug $x = 2$ into the function

$$3 \cdot 3^2 - 5 \cdot 3 - 2 = 0, \quad 3^2 - 5 \cdot 3 + 6 = 0$$

and hence both the numerator and denominator has a factor $x - 2$. Factoring gives us

$$2x^2 - 5x - 3 = (x - 2)(3x + 1)$$
$$x^2 - 7x + 12 = (x - 2)(x - 3)$$

and hence, for $x \neq 2, 3$,

$$\frac{3x^2 - 5x - 2}{x^2 - 5x + 6} = \frac{(x - 2)(3x + 1)}{(x - 2)(x - 3)} = \frac{3x + 1}{x - 3}.$$ 

In particular

$$\lim_{x \to 2} \frac{3x^2 - 5x - 2}{x^2 - 5x + 6} = \lim_{x \to 2} \frac{3x + 1}{x - 3} = \frac{3 \cdot 2 + 1}{2 - 3} = -7.$$ 

3. 2 marks Find the points of discontinuity of $f(x)$. For each point of discontinuity explain why the function is not continuous there. The function $f(x)$ is given by the graph.
The function is not continuous at

- $x = -7$ and $x = 6$ since the function is not defined near the point (to the left and to the right respectively) and hence the limit of the function at those points does not exist,
- $x = -3$ since the function is not defined there,
- $x = 2$ and $x = 4$ since the limit of $f(x)$ there does not exist and

**Long answer question — you must show your work**

4. **4 marks** Little Jonny is selling lemonade on Wesbrook Mall in summer. He is selling a cup for $1 and sells 100 cups a week. One week he tried to raise the price by 10 cents and he sold 20 cups less that week.

(a) Find the demand function linking $p$ and $q$.

**Answer:** $p = -\frac{1}{200}q + 1.5$ or $q = -200 \cdot p + 300$

**Solution:** Write $p = Aq + B$. Plugging in the data we have

\[
\begin{align*}
1 &= 100A + B \\
1.1 &= 80A + B
\end{align*}
\]

Solving for $A$ and $B$ yields

\[
A = -\frac{1}{200}, \quad B = 1.5.
\]

Alternatively, write $q = Cp + D$. Plugging the data yields

\[
\begin{align*}
100 &= C + D \\
80 &= 1.1 \cdot C + D
\end{align*}
\]

Solving for $C$ and $D$ yields

\[
C = -200, \quad D = 300
\]

(b) What is the price Jimmy needs to set for a cup of lemonade in order to maximize his revenue?

**Answer:** $£0.75$

**Solution:** We plug $q = -200 \cdot p + 300$ into $R = p \cdot q$ to get the revenue function

\[
R(p) = p \cdot (-200 \cdot p + 300)
\]
If \( R(p) = 0 \) then either \( p = 0 \) or \(-200 \cdot p + 300 = 0\), hence the two roots of the equation \( R(p) = 0 \) are \( p = 0 \) and \( p = \frac{3}{2} \). The maximal revenue will be the average of the two roots, namely \( p_{\text{max}} = \frac{0 + \frac{3}{2}}{2} = \frac{3}{4} \). 