1. The (minimum) weight matching problem given in the text can be solved using the associated minimum cost flow problem. The problem in the text concerned the complete graph on 8 vertices with edge weights given below (they appear as a symmetric matrix). I have listed the dual variables $\pi'_v$ and $\pi''_v$ for the minimum cost flow problem. Use this to determine a solution for the fractional matching problem. Determine the edges which have equality labelling and then by finding a minimum matching on this subgraph you will obtain the optimal matching. (I mentioned in class that this example in the text was not helpful because at optimality $\gamma_{S_k} = 0$ for all odd sets $S_k$.)

\[
\begin{bmatrix}
\pi'_v \\ \pi''_v
\end{bmatrix} =
\begin{bmatrix}
20 & 10 & 8 & 16 & 12 & 23 & 25 \\
19 & 8 & 8 & 18 & 18 & 25 & 29
\end{bmatrix}
\]

2. Consider a weighted matching problem which at optimality has no non zero $\gamma$ variables. Can you argue in general that this implies there is no need for the primal/dual algorithm but merely one run of a maximum cardinality matching problem in such a case.

3. Solve the following maximum weight matching problem using our primal-dual algorithm.

4. Prove the following results concerning Maximum Weight Spanning Trees of a connected graph $G = (V, E)$ with edge weights $w(e)$. These ideas are useful in algorithms.

a) If $w(i, j) = \max\{w(i, k) : (i, k) \in E\}$ then there is a maximum weight spanning tree containing the edge $(i, j)$.

b) If $C$ is a cycle of edges $e_1, e_2, \ldots, e_t$ and $w(e_i) = \min_{1 \leq i \leq t} w(e_i)$ then there is a maximum weight spanning tree not containing the edge $e_t$.

c) (From a talk of B. Chazelle) Let $U \subseteq V$ have the property that for every pair of edges $e_1 = (i, j), e_2 = (k, l)$ where $j, k \in U, i, l \not\in U$ there is a path $P$ from $j$ to $k$ of edges, entirely in $U$, so that the minimum edge weight in the path $P$ is at least $\min\{w(e_1), w(e_2)\}$. Then
there is a maximum weight spanning tree of $G$ which contains edges yielding a spanning tree on the vertices $U$. 