23. Let $G$ be a cubic simple connected planar map with 3 faces of size 4, $s$ faces of size 6 and $t$ faces of size 10. Determine $t$.

24. Classify which choices of $m$ and $n$ have the property that $K_{m,n} - e$ is planar (for any edge $e$ of $K_{m,n}$). Assume $m \geq n \geq 1$.

25. Given a planar (simple) graph $G$ show that the (edge) complement $G^c$ has the property that every vertex disjoint pair of odd cycles is joined by an edge. This problem arises in generating examples of graphs with this odd cycle property. It has been proven that if a graph $H$ has this odd cycle property and $H$ has a fractional $f$-factor where $\sum_i f_i$ is even then $H$ has an $f$-factor.

26. Let $G$ be a simple connected planar graph with all faces of even size. Show that $G$ is bipartite.

27. Show that the Petersen Graph is not planar using the Jordan Curve argument (in the same manner we used the Jordan Curve argument to show that $K_{3,3}$ and $K_5$ are not planar).

28. 
   a) Let $G$ be a simple graph with $\chi(G) = 3$. Show that there is a subset $S$ of the vertices with $|S| \geq (2/3)|V(G)|$ such that the subgraph of $G$ induced by the vertices of $S$ is bipartite.
   b) Extend this to graphs with $\chi(G) = k$ and show how to find a (surprisingly large) fraction of the vertices which induce a bipartite subgraph of $G$. Show the result is best possible for $G = K_n$.

29. Show that every graph $G$ has a subgraph $H$ which is both bipartite and has $|E(H)| \geq \frac{1}{2}|E(G)|$. Hint: The subgraph $H$ is spanning.