1. Let $G$ be a connected simple graph with $\Delta(G) = 3$ and $\delta(G) < 3$. Show that every subgraph of $G$ has a vertex of degree less than 3.

2. Let $G$ be a graph with exactly two vertices $x, y$ of odd degree and rest of even degree. Let $C$ be a cycle in $G$. Show that $G \setminus E(C)$ has an $x$-$y$-path.

3. Let $D$ be a directed graph on $n$ nodes with no directed cycles and with the property that for every pair of nodes $i, j$ we have either $i \to j$ or $j \to i$. Show that the nodes may be labelled $1, 2, \ldots, n$ so that $i \to j$ if and only if $i < j$.

4. Let $T$ be a tree. Let $\ell$ be the number of vertices of degree 1. Show that $\ell$ can be computed from the number of vertices of other degrees as follows:

$$\ell = 2 + \sum_{v : d(v) \geq 2} (d(v) - 2).$$

(The sum is over all $v$ such that $d(v) \geq 2$. )

5. Let $G$ be a bipartite graph with all vertices having even degree. Show that $G$ has a spanning subgraph $H$ with the property that for all $v \in V(G)$,

$$d_H(v) = \frac{1}{2} d_G(v).$$

6. Let $G$ be a bipartite graph with parts $X, Y$. Assume for each $A \subseteq X$, we have $|A| \leq |N(A)| - 1$. Show that $G$ has a matching of at least $|X| - 1$ edges.