problems are for classroom presentations and some fraction of them may be final exam questions.

23. Let $G$ be a simple connected planar graph with all faces of even size. Show that $G$ is bipartite.

24. Classify which choices of $m$ and $n$ have the property that $K_{m,n} - e$ is planar (for any edge $e$ of $K_{m,n}$). Assume $m \geq n \geq 1$.

25. Let $G$ be a cubic simple connected planar map with 3 faces of size 4, $s$ faces of size 6 and $t$ faces of size 10. Determine $t$.

26. Let $G$ be a planar graph. Show that we can find a planar drawing in which all but the outer face is a convex polygon. (recall our proof of Kuratowski’s Theorem)

27. 
   a) Let $G$ be a simple graph with $\chi(G) = 3$. Show that there is a subset $S$ of the vertices with $|S| \geq (2/3)|V(G)|$ such that the subgraph of $G$ induced by the vertices of $S$ is bipartite.
   b) Extend this to graphs with $\chi(G) = k$ and determine a large fraction of the vertices which induce a bipartite subgraph of $G$. Show the result is best possible for $G = K_n$. 