1. Use the notations $\delta(G)$ and $\Delta(G)$ for the minimum and maximum degrees.
   a) Show that $\delta(G) \leq \frac{2e(G)}{n(G)} \leq \Delta(G)$.
   b) Let $G$ be a connected graph with $\Delta(G) = 3$ and $\delta(G) < 3$. Show that every subgraph of $G$ has a vertex of degree less than 3.
2. The degree set of a graph is the set of degrees of the vertices of $G$. e.g. if every degree in $G$ is 3 then the degree set is $\{3\}$.
   a) Show that every set $S = \{a_1, a_2, \ldots, a_k\}$ for $k \geq 1$ of positive integers with $a_1 < a_2 < \cdots < a_k$ is the degree set of some graph.
   b) Find a graph of order 7 with degree set $S = \{4, 3, 5, 6\}$.
   c) Prove that if $u(S)$ is the minimum number of vertices in a graph with degree set $S$, then $u(S) = a_k + 1$.
3. Let $L(G)$ denote the line graph of $G$ namely $V(L(G)) = E(G)$ and $e_1e_2 \in E(L(G))$ if and only if the edges of $G$ $e_1, e_2$ have a common endpoint.
   a) Prove that if $G$ is eulerian, then $L(G)$ is eulerian.
   b) Give an example of a graph $G$ which is not eulerian and yet $L(G)$ is eulerian.
   c) Give a simple graph theoretic condition on $G$ so that $L(G)$ is eulerian.
4. Consider a graph with exactly two vertices $x, y$ of odd degree.
   a) Show that $G$ contains an $x$-$y$-path.
   b) If $G$ contains a cycle $C$ then $G \setminus E(C)$ contains an $x$-$y$-path.