

1. (warm-ups) 3.3.1, 3.3.2 and 3.1.28
2. Prove Hall's Theorem using the Max Flow-Min Cut Theorem. You will have to show that not all cuts are needed.
3. (adapted from 3.1.14) The Petersen graph has girth 5 and has no cycle of length 10.
 - a) Show that if G is the Petersen graph and M is a perfect matching in G then $G - M = C_5 + C_5$.
 - b) Prove that every edge in G lies in four 5-cycles, and count the 5-cycles in G .
 - c) Determine the number of perfect matchings in G
4. 3.3.20 Let G be a simple 3-regular graph.
 - a) Prove that if G has a decomposition into P_4 's then G has a perfect matching.
 - b) Prove that if G has a perfect matching then G has a decomposition into P_4 's
5. 3.3.6 Prove that a tree T has a perfect matching if and only if for each vertex $v \in V(T)$ $o(T - v) = 1$.
6. 3.3.16 Review proof of Petersen's Theorem 3.3.9 that a 3-regular graph with no cut edge has a 1-factor. Let G be a k -regular graph of even order that for any choice of $k - 2$ edges E' the subgraph $G - E'$ is connected. Prove that G has a 1-factor.