MATH 443 Assignment #2 Due Wednesday October 7.

1. 2.2.14 Let $f(r,s)$ be the number of trees with vertex set $[n]$ that have partite sets of sizes r and s (where $r+s=n$). Prove that $f(r,s)=\binom{r+s}{s}s^{r-1}r^{s-1}$ if $r\neq s$. (Hint: First show that the Prüfer sequence for such a tree will have $r-1$ of its terms from the partite set of size s and $s-1$ of its terms from the partite set of size r).

2. 2.2.21 Prove that $K_{2m-1,2m}$ decomposes into m spanning paths.

3. 2.2.22 Let G be a n-vertex simple graph that decomposes into k spanning trees. Given that also $\Delta(G)=\delta(G)+1$, determine the degree sequence of G in terms of n and k.

4. 2.2 33 Let T be an orientation of a tree such that the heads of the edges are all distinct; the one vertex that that is not a head is denoted the *root*. Prove that T is a union of directed paths from the root. Prove that for each vertex of T, exactly one path reaches it from the root.

5. Describe the possible forms of a directed graph which has one distinguished vertex r (denoted the *root*) with outdegree$(r)=0$ and for each vertex $v\neq r$, outdegree$(v)=1$.

6. 2.3.25 Let G be a tree where every vertex has degree 1 or degree k. Given k, for what values of $n(G)$ is this possible?

7. Consider a labeling of the edges of K_n by integer labels such that no two incident edges share a label. Show that there is a trail of $n-1$ edges consisting of edges $e_1,e_2,\ldots,e_{n-1}$ such that the labels are increasing (i.e. for $1\leq i<j\leq n-1$ the label of e_i is less than the label of e_j. (this is a MUCH harder problem so don't worry if you can't solve it)