MATH 443 Assignment #1 Due Wednesday September 23.

1. 1.1.38 A *claw* is a 4 vertex graph of 3 edges with one vertex joined to each of the other three. Let G be a simple graph with all vertices of degree 3. Prove that G has a decomposition into claws if and only if G is bipartite.

2. 1.2.33 Use induction on k or $|E(G)|$ to prove that a connected graph with exactly $2k$ odd degree vertices decomposes into k trails if $k > 0$. Does this remain true without the connectedness hypothesis?

3. 1.2.36 Prove that a connected graph is Eulerian if and only if each edge e is an odd number of cycles. You can no doubt prove this in many ways but here is the steps suggested in the text. This is a harder question. The second step is easier.

   First prove that if G is Eulerian and $uv \in E(G)$ then show that in $G' = G - uv$ that there are an odd number of u-v-trails that visit v only at the end. Prove also that the number of trails in this list that are not paths is even.

   Second Let v ve a vertex of odd degree in a graph. For each edge e incident to v, let $c(e)$ be the number of cycles containing e. Use $\sum_e c(e)$ to prove that $c(e)$ is even for some e incident to$v$.

   Now put these pieces together.

4. 1.2.40 Let P, Q be paths of maximum length in a connected graph G. Prove that P and Q have a common vertex.

5. 1.2.43 Use induction on $|E(G)|$ to show that every simple connected graph with an even number of edges ($|E(G)| \equiv 0 (\text{mod } 2)$) decomposes into paths of two edges (P_3's). Does the conclusion remain true if the hypothesis of connectedness is omitted?