Assume we are given say k points (x_1, y_1), $(x_2, y_2), \ldots, (x_k, y_k)$ corresponding to a piecewise linear curve $y = f(x)$ consisting of $k-1$ linear segments going through the k given points. For example the first line segment goes through the points (x_1, y_2) and (x_2, y_2). Using similar triangles (or your favorite procedure), if x satisfies $x_1 \leq x \leq x_2$, then there are multipliers w_1, w_2 with $x = w_1x_1 + w_2x_2$ where the multipliers satisfy $0 \leq w_1, w_2 \leq 1$ and $w_1 + w_2 = 1$. We conclude that the y value for this x value is $w_1y_1 + w_2y_2$. Thus we can substitute this equation for y (the variables are w_1, w_2) into an LP when we demand the equation $x = w_1x_1 + w_2x_2$ for $0 \leq w_1, w_2 \leq 1$ and $w_1 + w_2 = 1$.

To extend to k intervals, we create a total of $2k-1$ variables. First we have $k-1$ indicator variables that tell us which interval we are in.

$$d_i = \begin{cases} 1 & x \in (x_i, x_i + 1) \\ 0 & x \notin (x_i, x_i + 1) \end{cases}$$

I guess this leaves the boundary unclear. We do impose

$$d_1 + d_2 + \cdots + d_{k-1} = 1 \text{ and } d_i \in \{0, 1\}$$

and so for $x = x_i$ we allow either $d_{i-1} = 1$ or $d_i = 1$. We create k multipliers $w_1, w_2, \ldots, w_k$ with

$$x = w_1x_1 + w_2x_2 + \cdots + w_kx_k \text{ and } w_i\text{'s} \geq 0$$

$$w_1 + w_2 + \cdots + w_k = 1$$

and then we must ensure at most two multipliers are turned on so that for example if $x \in (x_1, x_2)$ then only w_1, w_2 can be non zero. These constraints are

$$w_1 \leq d_1,\ w_2 \leq d_1 + d_2,\ w_3 \leq d_2 + d_3,\ \ldots, w_{k-1} \leq d_{k-2} + d_{k-1}, w_k \leq d_{k-1}$$

Finally we have

$$y = w_1y_1 + w_2y_2 + \cdots + w_ky_k$$

This is a bit tricky. Our binary variables $d_1, d_2, \ldots, d_{k-1}$ satisfy $d_1 + d_2 + \cdots + d_{k-1} = 1$ and so exactly one d_i is nonzero, say $d_j = 1$. Then by the inequalities between the w_i's and d_i's, we have $w_1 = w_2 = \cdots = w_{j-1} = 0$ and $w_{j+2} = \cdots = w_{k-1} = w_k = 0$ and so $w_j + w_{j+1} = 1$. Now the equation $x = w_1x_1 + w_2x_2 + \cdots + w_kx_k$ reduces to $x = w_jx_j + w_{j+1}x_{j+1}$ and so x is the jth interval (x_j, x_{j+1}). This goes both ways, we have $d_j = 1$ if $x_j < x < x_{j+1}$).

Once we have x in the jth interval with $x = w_jx_j + w_{j+1}x_{j+1}$, then we conclude $y = w_jy_j + w_{j+1}y_{j+1}$ (by proportionality/similar triangles) and hence

$$y = w_1y_1 + w_2y_2 + \cdots w_ky_k.$$

This somewhat complicated sounding procedure works well in practice (google SOS2). It is probably not helpful to think of the effect on the simplex method pivots!