

1. Consider the problem of a UBC student about to graduate. The student needs to pass at least one of the three courses the student is enrolled in currently: German, Statistics, Chemistry. The student's busy schedule of extra-curricular activities leaves only 4 hours per week for studying. The student's probability of passing each course depends on the number of hours spent studying on the course. Use Dynamic Programming to determine how many hours per week the student should spend studying each course. (Hint: Explain why maximizing the probability of passing at least one course is the same as minimizing the probability of failing all three courses.)

Probability of Passing Courses

Hours of study per week	German	Statistics	Chemistry
0	.20	.25	.10
1	.30	.30	.30
2	.35	.33	.40
3	.38	.35	.45
4	.40	.38	.50

2. A Hugeburger franchise has just opened on Fraser Street. At present i_0 customers frequent Hugeburger (we call these loyal customers) and $N - i_0$ customers frequent other fast food establishments (we call these non-loyal customers). At the beginning of each month, Hugeburger must decide how much to spend on promotional materials. At the end of the month in which Hugeburger spends \$ d on advertising, a fraction $q(d)$ of the non-loyal customers become loyal and a fraction $p(d)$ of the loyal customers become non-loyal. During the next 12 months Hugeburger wishes to spend \$ D on promotion. You should imagine the budget is \$20,000 and you are going to spend it in \$1000 chunks. Develop a Dynamic Programming formulation that will enable Hugeburger to maximize the number of loyal-customers the company will have at the end of month 12 month (ignore problems like fractional customers!). You find going forward in time will be natural in this problem. Try to formulate in the format of Bellman's equation but of course it will be quite different since you will be counting the number of customers. You don't have to solve since you don't have the data.

3. Consider the following problem with stochastic demand. The retailer does not know demand with certainty. The demand for the item is the same for each month and is given by the following distribution

Demand	probability
10	0.2
20	0.5
30	0.3

The retailer orders at the beginning of each month. Initially he has no units of the item in stock. It costs \$20 to place an order. Each unit costs \$4 and is sold for \$6. Each unit of demand in excess of the units on hand is lost and incurs a penalty of \$2 per unit of unmet demand. The retailer can only order in lots of 10,20,30,... units and that the maximum he can order in any month is 60 units. Further assume that he receives the order immediately (no lead time) and the demand occurs after the order. Any units left over at the end of a month will be transferred to the next month but at an inventory cost of \$.10 per unit. The inventory cost is charged to the month receiving the units. He attempts to stock whatever remains at the end of a month but cannot store more than 40 units

- units in excess of 40 are discarded at no additional cost and no special salvage value.

How many units should be ordered each month to minimize total expected net cost over a planning horizon of 12 months? Sorry, profit appears as a negative.

a) Outline a dynamic programming formulation. I've included the spreadsheet for the first two months from the end (two months to go). Check out the spreadsheet on asst6retailer.xls on sheet 2. Indicate how the strange formulas for 'two months to go' with $s_2 = 30$ and $d_2 = 20$ correspond to your formulation. For example \$I\$2, \$I\$3, \$I\$4 are the probabilities given near the top of the spreadsheet.

b) You can 'copy' the table further down in your spreadsheet to get the table for three periods to go (but you would have to update the labels at the top of the table such as replacing d_2 by d_3). Compute from your tables the optimal decisions given that you are three months to go and have a stock of 20 units (on which you are about to charge inventory). Give decision trees.

I've included in the spreadsheet for the first two calculations on asst6retailer.xls on sheet 2 (click on sheet 2 tab at bottom). Note that the entries for $d_i^*(s_i)$ are computed by hand. In Excel the entries are indexed by column (a letter) and row (a number) so that A5 refers to column A and row 5. One can enter numbers into entries or formulas which evaluate to a number. Entering a formula has a few tricks. Click = to enter a formula, an expression involving other entries that can use special function such as Max or Min or have logical expressions such as $IF(X > 0, 20)$ which returns 20 if $X > 0$. At this point excel would seem to be no better than a cumbersome calculator. But you can copy an entry or table of entries and paste into other cells. When the cell you are copying has a formula then the formula is updated as follows. If a formula refers to entry A5 and you copy the cell 5 rows down and two columns to the left then the entry A5 in the formula is replaced by C10. Thus you can take a given cell and copy it into a range of cells each acquiring a 'new' formula. You can adjust this by using \$ signs. Thus when you copy \$A5 5 rows down and two columns to the left then the entry \$A5 in the formula is replaced by \$A10, when you copy A\$5 5 rows down and two columns to the left then the entry A\$5 in the formula is replaced by C\$5 and when you copy \$A\$5 5 rows down and two columns to the left then the entry \$A\$5 in the formula is replaced by \$A\$5. Now when a formula is evaluated to a number the \$'s are ignored.

c) Complete the calculations for twelve months (no need for decision trees!). Consider the tables with 12,11,10 months to go and outline the general pattern you see in terms of retailers optimal decisions (you will have to compute the optimal decision for each value of s_i). Can you give an explanation why the profit per period for the retailer appears to be approximately \$29 per month, or so it appears from these tables? Use the notation

$$\mathbf{v}_i = \begin{bmatrix} v_i(0) \\ v_i(10) \\ v_i(20) \\ v_i(30) \\ v_i(40) \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 94 \\ 55 \\ -100 \\ -123 \\ -122 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & .3 & .5 & .2 \\ 0 & 0 & .3 & .5 & .2 \\ .8 & .2 & 0 & 0 & 0 \\ .3 & .5 & .2 & 0 & 0 \\ 0 & .3 & .5 & .2 & 0 \end{bmatrix}$$

Explain why we have the following recurrence when the decisions are those you have established with at least 5 months to go : $\mathbf{v}_{i+1} = \mathbf{b} + A\mathbf{v}_i$. Now show

$$\mathbf{v}_{i+k} = \mathbf{b} + A\mathbf{b} + A^2\mathbf{b} + \dots + A^{k-1}\mathbf{b} + A^k\mathbf{v}_i$$

and show (using some matrix theory and eigenvector/eigenvalue calculations) that

$$v_{i+1}(s_i) \approx v_i(s_i) - 29.$$