

A reminder to print out your formulation and relevant output. Of course the answer should also be given in terms of the original problem is also essential. The grader will be instructed to mark down assignments which merely turn in output and do not explain their formulation (e.g. what do the variables represent) and the output (for this problem a weekly work schedule).

1 (Some computational aspects of Integer Linear Programs). A bus company requires different numbers of employees on different days of the week:

Monday: 18, Tuesday: 16, Wednesday: 16, Thursday: 17,

Friday: 20, Saturday: 14, Sunday: 8.

Union rules states that each full-time employee must work 5 consecutive days and then receive two days off. For example, an employee who works Monday to Friday must have Saturday and Sunday off.

- a) The Bus company wants to meet its daily requirements only using full-time employees. Formulate the problem of minimizing the number of full-time employees (and hence the wage costs) as a linear programming problem and solve using LINDO without imposing integrality. Rerun using the integer specification GIN and compare the solutions.
 - b) Consider a variation on a) where we allow part-time workers and overtime. We allow part-time workers who work half days for 5 consecutive days but at .7 the wage rate of full-time employees because of reduced benefits. That is a part-time worker is paid .35 the wage rate of a full-time worker in a week. Union rules specify that at most 25% of the work can be done by part-timers (otherwise, because of reduced pay, only part-timers would be hired). Be careful with this inequality. I mean that paid part-time work only accounts for 25% of the total paid work. Also allow the possibility of hiring full-time employees on overtime to work an extra half day on their day off at 1.8 the regular wage of full-time employees. Seek a solution (with integer variables) whose cost is $21.03 \times$ the cost of a full-time employee. Such a solution will be obtained relatively quickly but proving optimality by the software will take a while. Comment on the difficulty of obtaining verification of optimality using LINDO or other integer linear programming software (some iteration count should be recorded).
 - c) Justify that an optimal solution will have at most 11 part-time workers (for this particular problem data). Add this constraint to the ILP in b) and as well add a constraint that your solution has value at most 21.03 (why can you do this?) and rerun. Hopefully the iteration count will have miraculously dropped (it did for me). Comment.
 - d) You may recall that the last bus strike was over part-time workers (and split shifts). For this simplified model, what is the value of having part-time workers? If part-time workers were available but at the full wage rate (per hour) of full-time workers, would they still result in savings (for the bus company)?
2. (Piecewise Linear Functions and using LINGO) Consider the hiring/firing model. Indicate how to add the constraint that you can fire as many workers as you want in a month but the cost increases with a cost of \$420 for first 10, \$600 for next 20 and \$2000 for any workers beyond 30 in a given month. Alter the existing LINGO model (on the website) and solve (both obtaining a fractional solution and also the integer solution) and obtain the integrality gap (by comparing the two objective functions). My answer was \$173,400 with an integrality gap of 0.
3. (Branch and Bound) Use our branch and bound procedure to solve the following LP where

$$\begin{array}{rcccccc}
\max & 3x_1 & +6x_2 & +25x_3 & +60x_4 & = z \\
& 24x_1 & +76x_2 & +43x_3 & +754x_4 & \leq 910 - n \\
LP = & 755x_1 & +27x_2 & +33x_3 & +67x_4 & \leq 910 - n \\
& x_1 & & & & \leq 1 \\
& & x_2 & & & \leq 1 \\
& & & x_3 & & \leq 1 \\
& & & & x_4 & \leq 1
\end{array}
\quad x_1, x_2, x_3, x_4 \geq 0.$$

where n is the number formed from the last two digits of your student number. For the student whose initials are A.E.A., please use $n = 81$.

Consider the ILP (Integer Linear Program) obtained from the above LP by adding the constraints that all the variables are 0 or 1. Use Branch and Bound to solve. You may make your own choices which variables to branch on. Record for each LP solved the value for z , the solution and as well as the extra constraints you have added to the LP which determine the current LP. Obtain a Branch and Bound tree as given in the handout with the actual solutions recorded.

Now obtain a solution to the ILP above by using LINDO with the GIN command (INT will also work since the variables are 0 or 1). Obviously the final answer, at least its z values should agree with what you obtained by using Branch and Bound. Record number of iterations. You might also try to figure out what LINDO is doing (it appears to be our branch and bound with something else I don't understand!).