1 The final dictionary is

\[
\begin{align*}
x_3 & = 8 -3x_6 +x_2 +x_4 \\
x_5 & = 11 -5x_6 +3x_2 +2x_4 \\
x_1 & = 2 -x_6 \\
z & = 2 -2x_6 -x_4
\end{align*}
\]

2. \((y^*_1, y^*_2, y^*_3) = (0, 3, 4)\)

3. \(c^*_N - c^*B^{-1}A_N = \begin{bmatrix} 1 & 2 & -1 & -1 \end{bmatrix}\) so \(x_2\) enters and \(x_3\) leaves.

4a) \((2,1,0)\)

4b) yes marginal cost < profit

4c) \(5 \leq c_3 \leq 17/3\)

4d) \(13 \leq b_2 \leq 31/2\)

4e) new solution \(x_5 = 1, x_2 = 0, x_3 = 1\) with marginal values \((3, 0, 1/2)\)

4f) new solution \(x_1 = 1/3, x_2 = 5/3, x_3 = 2, x_4 = 1/3\) with marginal values \((0, 5/3, 0, 2/3)\) where last entry is marginal value from new constraint.

5a) 1 is an allowable decrease so the tons of carbon removed drops by 1.5

5b) carbon removed increases by 13 tons.

5c) cost of strategy/tons = 42500/113.5 <$400

5d) marginal cost of new strategy is 26 which is greater than 19 (value in tons removed) so no interest in this strategy

5e) 31.5 fewer tons of carbon removed

6. Objective function measures the payoff to player 1 (row player) if player 2 (the column player) plays strategy \(y\)

7. The following primal/dual pair is crucial.

\[
\begin{align*}
\text{max } & \quad 0 \cdot x \\
\text{min } & \quad b \cdot y + d \cdot z \\
Ax & \geq b \\
Cx & = d \\
y & \geq 0
\end{align*}
\]

8. Use convexity on vector \(c\).

9. Using strategy for player 2 (column player) is \(y = (1/16, 1/16, \ldots, 1/16)^T\) we obtain \(v(A) \leq 12/16\). Using strategy for player 1 (row player), that gives a weight of 1/8 to each of 8 dominoes which cover the \(4 \times 4\) board, we get \(v(A) \geq 6/8\). And so \(v(A) = 3/4\).
1. The final dictionary is

\[ \begin{align*}
x_2 &= 3 + 2x_1 + x_4 \\
x_3 &= 1 + x_1 + x_4 - x_5 \\
x_6 &= 2 + 5x_1 + 3x_4 - x_5 \\
z &= -2 - x_5
\end{align*} \]

2. \((y_1^*, y_2^*, y_3^*) = (2, 1, 0)\)

3. \(c_N^r - c_B^r B^{-1} A_N = \begin{bmatrix} x_1 & x_3 & x_6 & x_7 \end{bmatrix} \) so \(x_1\) enters and \(x_2\) leaves.

4a) \((2, 1, 0)\)
4b) \(c_2 \leq 7\)
4c) \(2 \leq c_3 \leq 4\)
4d) \(-1 \leq p \leq 1\) and profit is \(7 + 3p\).
4e) new solution \(x_1 = 1/2, x_3 = 3/2, x_2 = 1/2\) with marginal values \((1, 5/4, 1/4)\)
4f) new solution \(x_1 = 1/2, x_3 = 1/2, x_6 = 4, x_2 = 1/2\) with marginal values \((2, 1/2, 0, 1/2)\) where last entry is marginal value from new constraint.

5a) Increase for service 3 by 1 results in profit increase by 6.25
5b) change is in allowable range so profit changes by \(-35\) (i.e. drops).
5c) \(\max 20x_1 + 25x_2 + 38x_3 - 5d_1 - 5d_2 - 5d_3 - 4e_3 = 50\)
5d) \(\min 20x_1 + 25x_2 + 38x_3 - 5d_1 - 5d_2 - 5d_3 - 4e_3 = 50\)
5e) Now the dual prices are \((-5, -0.8333, 6.25)\)

6a) Now \(\mathbf{1} \cdot \mathbf{x} = x_1 + x_2 + \cdots + x_n\). If \(\mathbf{x} = \mathbf{0}\), then \(\mathbf{1} \cdot \mathbf{x} = 0\). Thus if \(\mathbf{1} \cdot \mathbf{x} \neq 0\) then \(\mathbf{x} \neq \mathbf{0}\).
6b) The following primal/dual pair is crucial.

\[
\begin{align*}
\text{max} & \quad \mathbf{1} \cdot \mathbf{x} \\
\text{min} & \quad \mathbf{0} \cdot \mathbf{y} \\
A\mathbf{x} & = \mathbf{0} \\
\mathbf{x} & \geq \mathbf{0} \\
A^T\mathbf{y} & \geq \mathbf{1} \\
\mathbf{y} & \text{ free}
\end{align*}
\]

7. This is like an assignment question. You can note that the LP is feasible since \(\mathbf{x} = \mathbf{0}\) is feasible. Then you must show LP is bounded and you can show that each variable is bounded.
8a) Given a strategy \(\mathbf{x}\) with \(x_k > 0\), then we can obtain a new mixed strategy \(\mathbf{x}^*\) at least as good by setting \(x_k^* = 0\) and \(x_\ell^* = x_\ell + x_k\).
8b) Follow the above idea and check that \(\mathbf{x}^T A < (\mathbf{x}^*)^T A\). Thus \(\mathbf{x}\), with \(x_k > 0\), was never an optimal strategy.
9. This is much like the assignment 4 question 3.