This result is due to Bob Bland of Cornell University. Bland’s Rule is the pivot rule that is the same as Anstee’s Rule except that when choosing an entering variable we choose the variable of smallest subscript among all those with a positive coefficient in the z row.

**Theorem** If we use Bland’s Rule, we will not cycle. 

**Proof:** Assume we do have cycling using Bland’s rule with $B_1 \rightarrow B_2 \rightarrow B_3 \rightarrow \cdots \rightarrow B_\ell \rightarrow B_1$. Let $D_i$ be the dictionary associated with basis $B_i$. Let a variable $x_j$ be called fickle if $x_j \in B_g$ and yet $x_j \notin B_h$ for some pair $g, h \in \{1, 2, \ldots, \ell\}$. We can describe the fickle variables as precisely those either entering or leaving during the cycle. We already know in a cycle that each pivot will be a degenerate pivot (you should know why?) and hence, if $x_j$ is fickle, $x_j = 0$ in each basic feasible solution during the cycle.

Let $x_t$ be the fickle variable of largest subscript. Assume $x_t \in B_i$ and yet $x_t \notin B_{i+1}$. Assume $x_s$ is the entering variable as we pass from $B_i$ to $B_{i+1}$. Thus $s < t$. We write the dictionary $D_i$ for $B_i$ as follows

$$D_i \begin{cases} x_k &= b_k - \sum_{x_j \notin B_i} a_{kj} x_j \\ z &= v + \sum_{x_j \notin B_i} c_j x_j \end{cases}$$

We have chosen useful terminology. We are not asserting that $a_{kj}$ nor $c_j$ arises from the original data. Since we chose $x_s$ to enter we have $c_s > 0$. Now $x_t$ leaving means that $a_{ts} > 0$ and $b_t = 0$. Now assume $x_t$ reenters basis for the first time in the pivot from $D_f$.

$$D_f \begin{cases} \cdots &= \cdots \\ z &= v + \sum_{x_j \notin B_f} c_j^* x_j \end{cases}$$

Again we have chosen useful terminology and are not asserting that somehow $c_j^*$ is optimal, which wouldn’t make much sense anyway. We deduce that $c_t^* > 0$. The equation for $z$ in $D_f$ is obtained from the equation from $D_i$ by adding suitable multiples of equations of $D_i$ to the $z$ row. A solution of $D_i$ which is not necessarily feasible is

$$x_s = q \\
x_j = 0 \text{ for } x_j \notin B_i \text{ and } j \neq s \\
x_k = b_k - a_{ks} q \text{ for } x_k \in B_i \\
z = v + c_s q$$

As we showed early on, the set of solutions is preserved as we move from dictionary to dictionary (we are multiplying the set of equations on the left by an invertible matrix) and so these solution (one for each $q$) are also solutions to $D_f$. Thus

$$z = v + c_s q = v + c_s^* q + \sum_{x_k \in B_i} c_k^*(b_k - a_{ks} q)$$

where we set $c_k^* = 0$ for $x_k \in B_f$. We now have

$$\left( c_s - c_s^* + \sum_{x_k \in B_i} c_k^* a_{ks} \right) q = \sum_{x_k \in B_i} c_k^* b_k.$$
This above equation is true for all \( q \) and so we deduce

\[
c_s - c_s^* + \sum_{x_k \in B_i} c_k^* a_{ks} = 0
\]

Because \( x_s \) is not entering in \( D_f \), we have \( c_s^* \leq 0 \) (if \( c_s^* > 0 \), then \( x_s \) would be chosen over \( x_t \) as the entering variable by Bland’s Rule). Hence

\[
\sum_{x_k \in B_i} c_k^* a_{ks} < 0
\]

and so for some \( r \) with \( x_r \in B_i \), we will have

\[
c_r^* a_{rs} < 0.
\]

Thus \( c_r^* \neq 0 \) and so \( x_r \notin B_f \) and hence \( x_r \) is fickle. Now we have already found \( c_t^* a_{ts} > 0 \) and so \( r \neq t \) and so by our choice of \( x_t \) we have \( r < t \). Also \( x_r \) is not the entering variable in \( D_f \) and so \( c_r^* \leq 0 \). Thus \( a_{rs} > 0 \). (We also obtain \( c_r^* < 0 \)). Now because \( x_r \) is fickle, we deduce that \( b_r = 0 \) in \( D_i \). But now when we pivot in \( D_i \) we would preferentially choose \( x_r \) over \( x_t \) as the leaving variable, a contradiction. Thus cycling does not occur while using Bland’s Rule.